A Controlled Mean Field Model for Chiplet Population Dynamics

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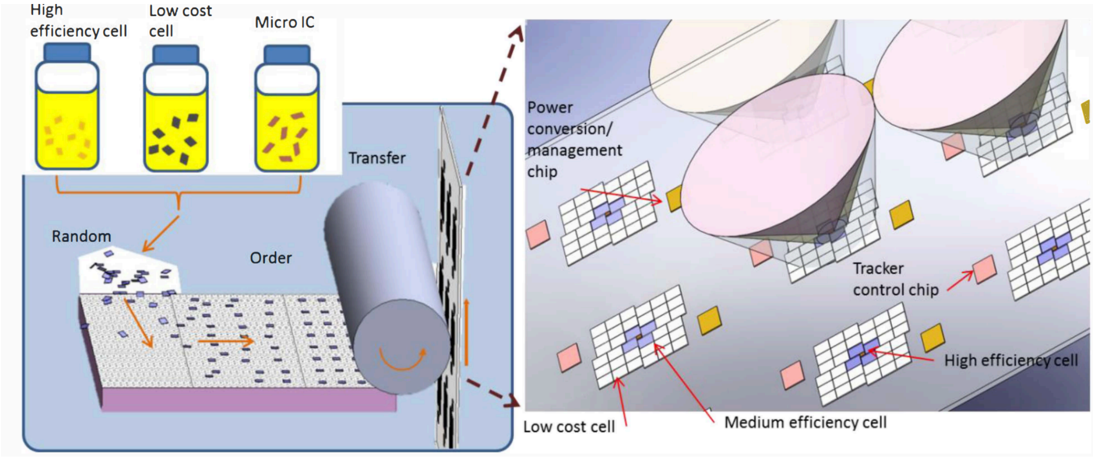
Topic of this talk

Model dynamics of "chiplet population": large ensemble of micro/nano sized particles immersed in dielectric fluid

Motivating applications

Xerographic micro-assembly for printer systems

Manufacturing of photovoltaic solar cells



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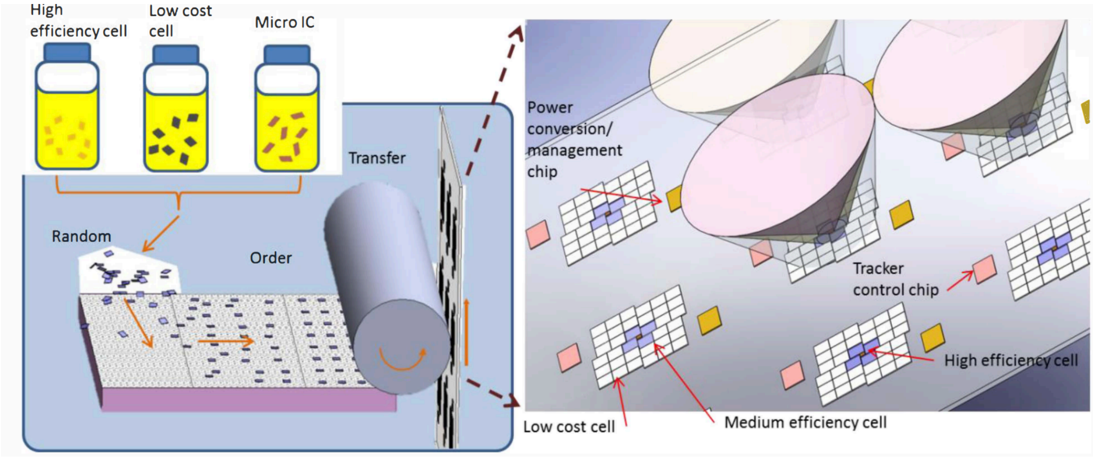

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Actuation and control

Electric potential generated by very large array of small electrodes

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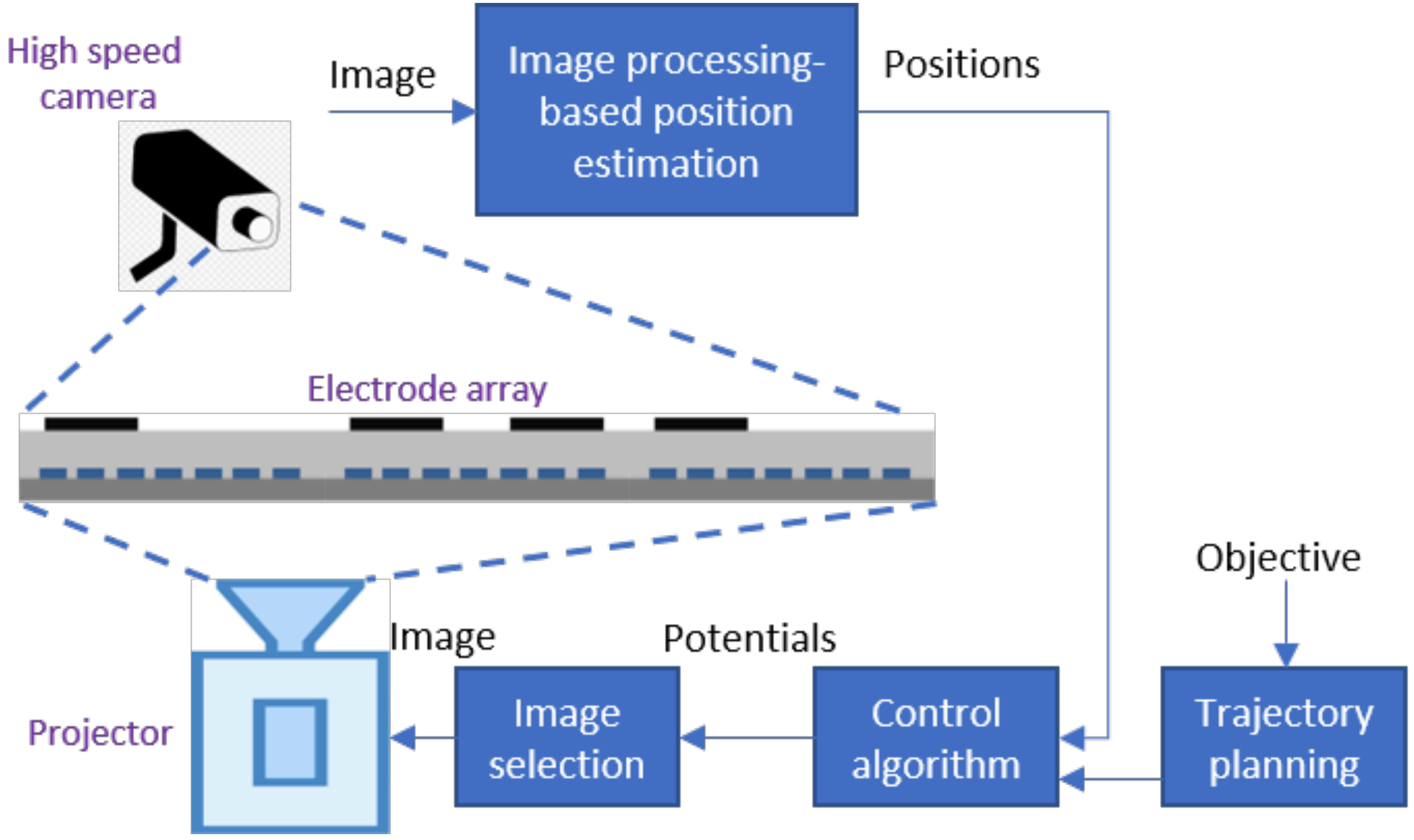
Spatio-temporally non-uniform dielectrophoretic forces on the chiplets

Typical experimental setup



Existing state-of-the-art

Several works on modeling the finite population:

[Lu *et. al.*, *Appl. Phys. Lett.*, 2014]

[Edward and Bevan, *Langmuir*, 2014]

[Matei *et. al.*, *CDC*, 2020]

[Matei *et. al.*, *CDC*, 2021]

[Lefevre *et. al.*, *IEEE/ASME Trans. on Mechatronics*, 2022]

How to steer the large finite population toward desired pattern:

Vectorize the positions of all chiplets, then apply MPC [Matei *et. al.*, *US Patent* 17121411]

Computation does not scale … need new ideas

Main idea

What we want to control is population-valued trajectory … not a finite dim signal

Derive continuum model and design optimal control in that limit

Then apply that optimal control to large but finite population

This work: only the first step: derive controlled dynamics in the limit both # electrodes and # chiplets $\to \infty$

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Technical challenge: two types of Coulomb interactions

1. Chiplet-to-chiplet interaction

2. Chiplet-to-electrode interactions

Both interactions are nonlinear in state + non-affine in control input

Derived model

2D position of an individual chiplet: $\boldsymbol{x}(t) \in \mathbb{R}^2$

Causal deterministic control policy: $u : \mathbb{R}^2 \times [0, \infty) \mapsto [u_{\min}, u_{\max}] \subset \mathbb{R}$

Electric voltage (u) — Typically [-400, 400] Volt ($[u_{\min}, u_{\max}]$)

At low Reynold's number in dielectric fluid (ignoring small mass of chiplet):

$$\underbrace{\mu \dot{\boldsymbol{x}}}_{\text{viscous drag force}} = \underbrace{\boldsymbol{f}^u}_{\text{controlled interaction force}} + \text{noise}$$

At time *t*, normalized chiplet population density function (PDF): $\rho(\boldsymbol{x}, t) \in \mathcal{P}_2\left(\mathbb{R}^2\right)$

The vector field: $\boldsymbol{f}^u : \mathbb{R}^2 \times [0, \infty) \times \mathcal{U} \times \mathcal{P}_2(\mathbb{R}^2) \mapsto \mathbb{R}^2$

Derived model: nonlocal Itô SDE

W.l.o.g. viscous coefficient $\mu = 1$ (else re-scale vector field)

Itô SDE for the i th chiplet:

$$\mathrm{d}\boldsymbol{x}_i = \boldsymbol{f}^u\left(\boldsymbol{x}_i, t, u, \rho^n\right) \mathrm{d}t + \sqrt{2\beta^{-1}}\, \mathrm{d}\boldsymbol{w}_i(t) \quad \text{with i.i.d.} \quad \boldsymbol{x}_{0i} \sim \rho_0 \in \mathcal{P}_2\left(\mathbb{R}^2\right) \quad \forall\, i \in [\![n]\!],$$

$\rho^n := \frac{1}{n} \sum_{i=1}^n \delta_{\boldsymbol{x}_i}$

Standard Wiener process

Non-local vector field:

$$\boldsymbol{f}^u(\boldsymbol{x}, t, u, \rho) = -\nabla\left(\int_{\mathbb{R}^2} \phi^u(\boldsymbol{x}, \boldsymbol{y}, t)\rho(\boldsymbol{y}, t)\mathrm{d}\boldsymbol{y}\right) = -\nabla\left(\rho * \phi^u\right)$$

Controlled interaction potential Comma … not minus Generalized convolution

Derived model: controlled interaction potential ϕ^u

Non-local vector field:

$$f^u(x, t, u, \rho) = -\nabla \left(\int_{\mathbb{R}^2} \phi^u(x, y, t) \rho(y, t) \mathrm{d}y \right)$$

Controlled interaction potential $= \phi^u_{\mathrm{cc}}(x, y, t) + \phi^u_{\mathrm{ce}}(x, y, t)$

$$\phi^u_{\mathrm{cc}}(x, y, t) := C_{\mathrm{cc}}(\|x - y\|_2)(\bar{u}(y, t) - \bar{u}(x, t))^2/2,$$
$$\phi^u_{\mathrm{ce}}(x, y, t) := C_{\mathrm{ce}}(\|x - y\|_2)(u(y, t) - \bar{u}(x, t))^2/2,$$

Capacitances (in practice, from COMSOL electrostatic simulation)

$$\bar{u}(x, t) := \frac{\int_{\mathbb{R}^2} C_{\mathrm{ce}}(\|x - y\|_2) u(y, t) \rho(y, t) \mathrm{d}y}{\int_{\mathbb{R}^2} C_{\mathrm{ce}}(\|x - y\|_2) \rho(y, t) \mathrm{d}y}.$$

Consistency guarantee for the mean field limit

Thm. (informal)

The random empirical measure $\rho^n \rightharpoonup \rho$ a.s. in the limit $n \uparrow \infty$

where ρ solves the nonlinear McKean-Vlasov-Fokker-Planck-Kolmogorov IVP

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \boldsymbol{f}^u) + \beta^{-1} \Delta \rho \\ &= \nabla \cdot \left(\rho \nabla \left(\rho * \phi^u + \beta^{-1}(1 + \log \rho)\right)\right),\end{aligned}$$

$$\rho(\cdot, t = 0) = \rho_0 \in \mathcal{P}\left(\mathbb{R}^2\right) \text{ (given).}$$

Chiplet mean field dynamics as Wass. grad flow

Thm. (informal)

Define "energy functional" $\Phi(\rho) := \mathbb{E}_\rho\left[\rho * \phi^u + \beta^{-1} \log \rho\right]$

Then

(i) $\dfrac{\partial \rho}{\partial t} = -\nabla^W \Phi(\rho).$

(ii) $\Phi(\cdot)$ is a Lyapunov functional for the mean field dynamics.

Wasserstein proximal recursion

Thm. (informal)

Let

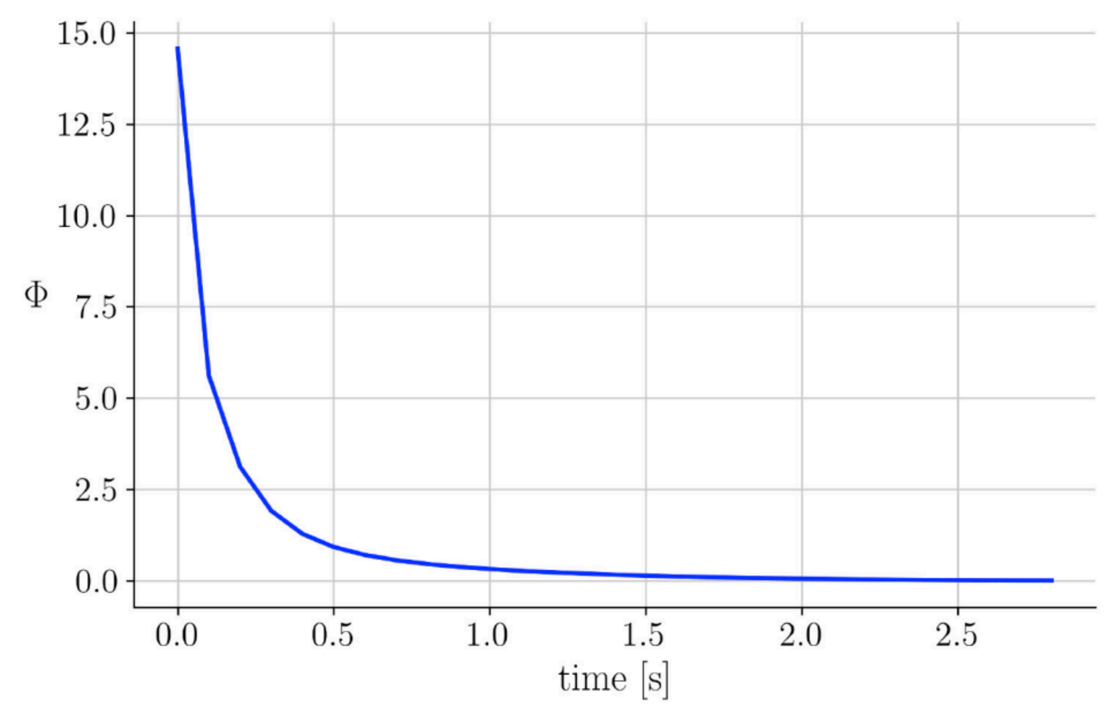
$$\widehat{\Phi}(\varrho, \varrho_{k-1}) := \mathbb{E}_{\varrho}\left[\varrho_{k-1} * \phi^{u} + \beta^{-1}\log\varrho\right], \varrho, \varrho_{k-1} \in \mathcal{P}_2\left(\mathbb{R}^2\right)\ \forall k \in \mathbb{N}$$

Then the proximal recursion

$$\begin{aligned}\varrho_k &= \mathrm{prox}^{W}_{\tau\widehat{\Phi}}(\varrho_{k-1}) \\ &:= \underset{\varrho\in\mathcal{P}_2(\mathbb{R}^2)}{\arg\inf}\left\{\frac{1}{2}W^2(\varrho, \varrho_{k-1}) + \tau\,\widehat{\Phi}(\varrho, \varrho_{k-1})\right\}\end{aligned}$$

approximates the transient solutions of the mean field nonlinear PDE IVP

Can be used for forward simulation (instead of finite difference etc.)



Summary of contributions

Derived a controlled mean field model for chiplet population dynamics

Recast it as Wasserstein gradient flow, and Wasserstein proximal recursion

Ongoing efforts

Optimal control synthesis

subject to the mean field dynamics + endpoint constraints

Thank You

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