Schrödinger Bridges in Learning and Control

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Joint work with students and collaborators

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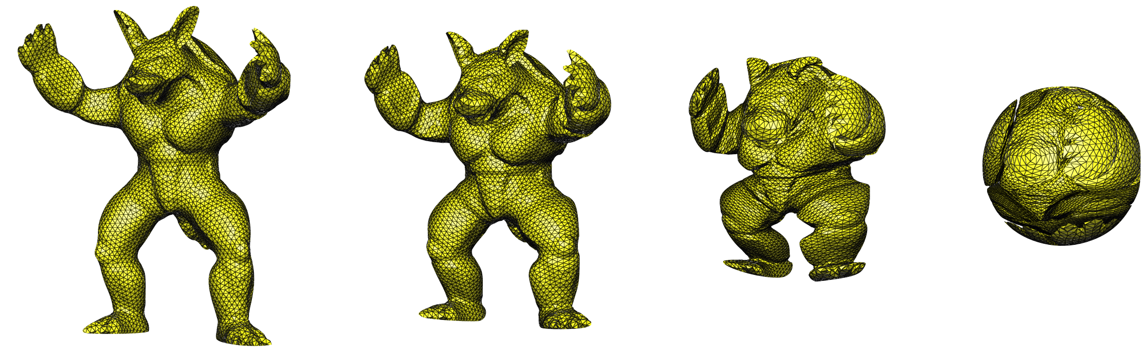
Bridge ⇝ Transport from Source to Target

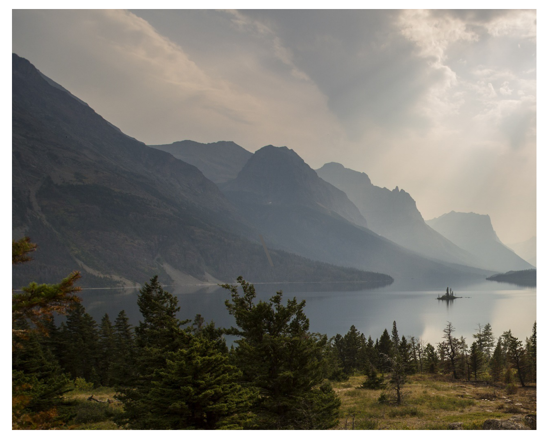
of goods

of meshes



of color

source



target

transported

of style

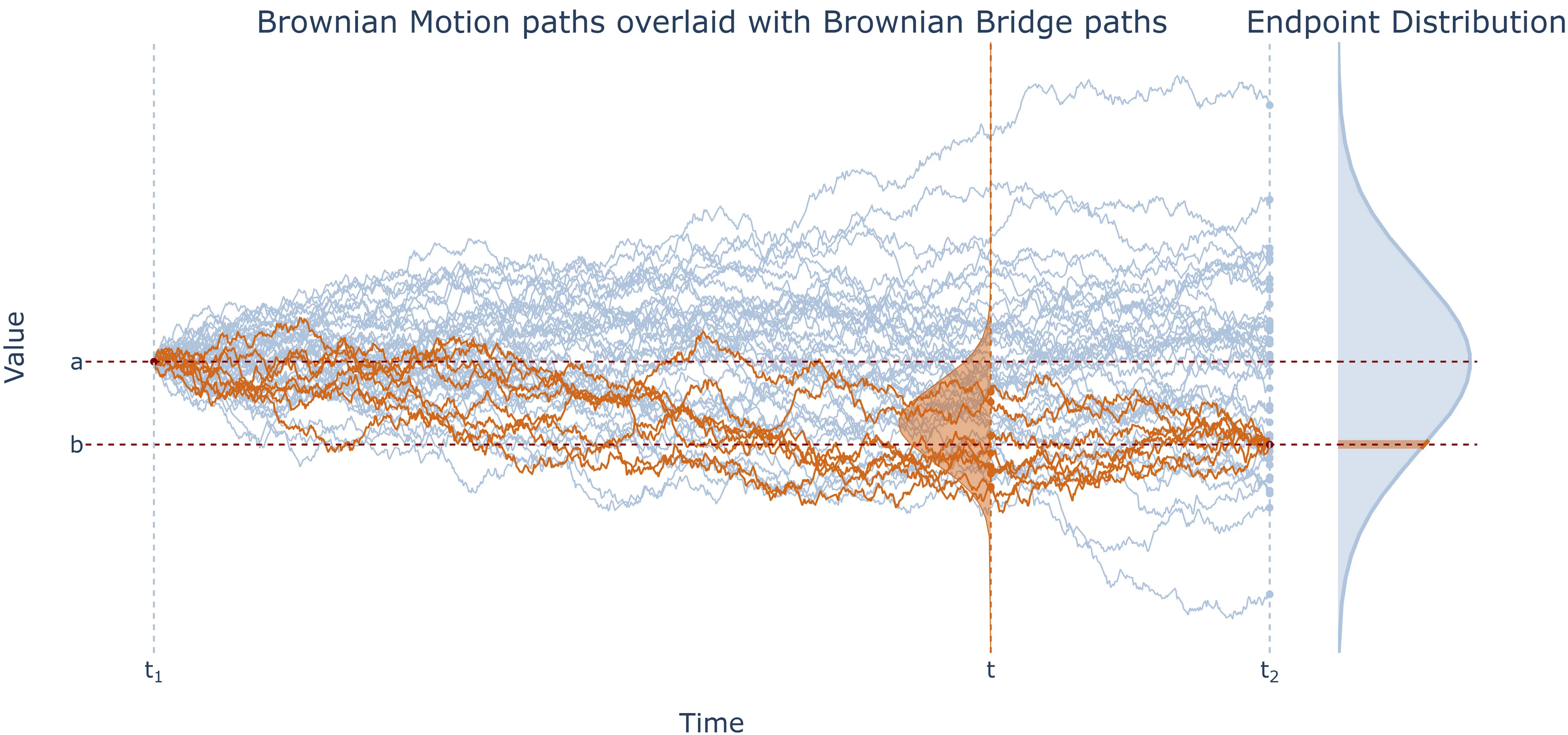
source

target

transported

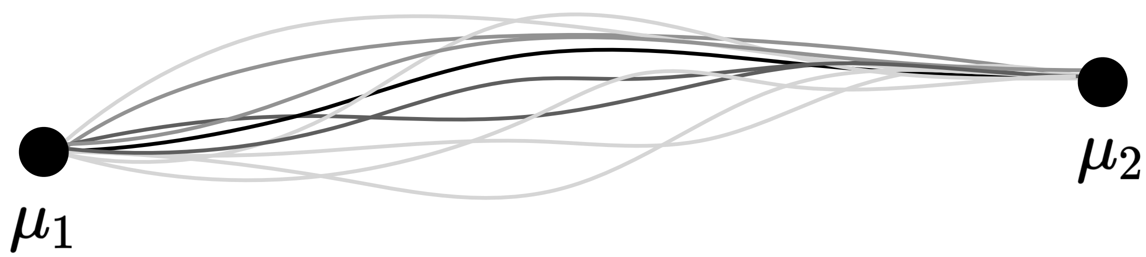
Diffusion Bridge: Transport via Diffusion

When source and target are points on the ground vector space

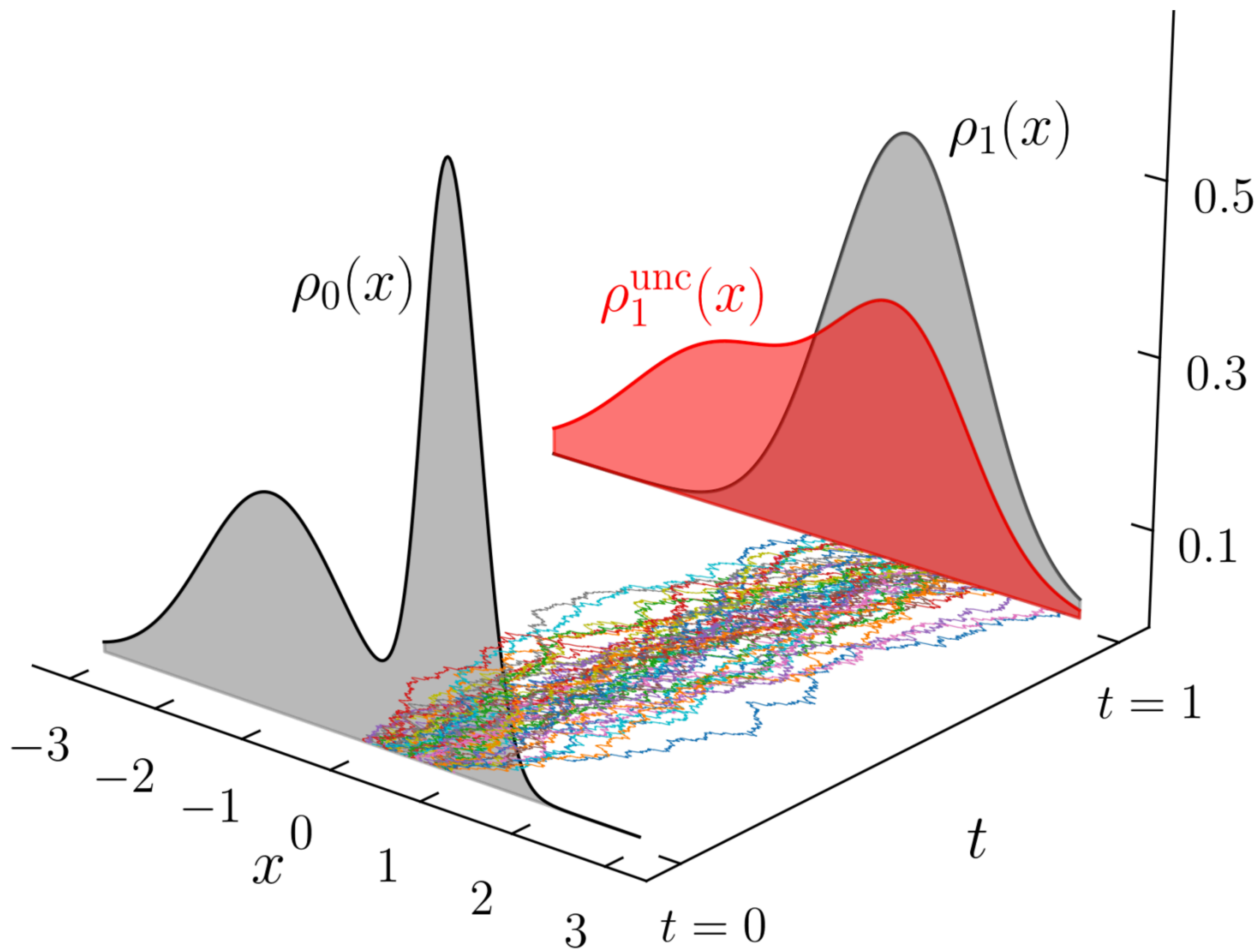


Schrödinger Bridge (SB): Transport via Diffusion

When source and target are measures or probability density functions (PDFs)

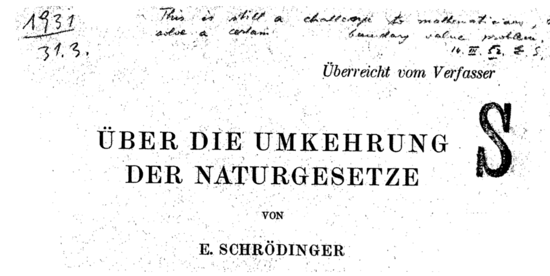


Comes with maximum likelihood guarantee on path space



Brief History of SB

formulates the problem as an attempt to give stochastic interpretation of quantum mechanics

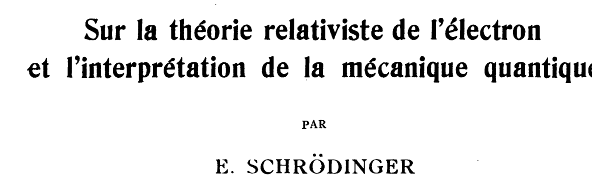


Überreicht vom Verfasser

ÜBER DIE UMKEHRUNG
DER NATURGESETZE

VON

E. SCHRÖDINGER



Sur la théorie relativiste de l'électron
et l'interprétation de la mécanique quantique

PAR

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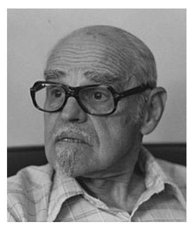
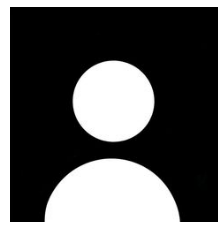
E. Schrödinger (1931-32)

R. Fortet (1940)



A. Beurling (1960)

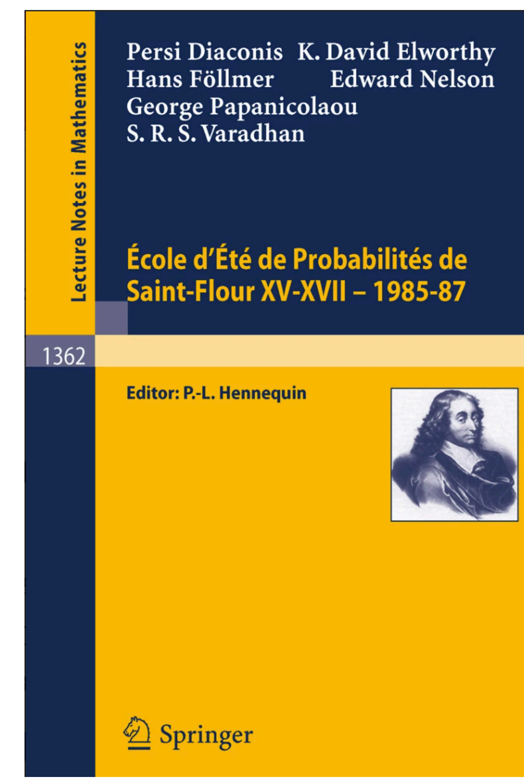


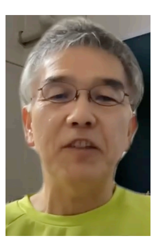
B. Jamison (1975)

establish the existence-uniqueness of the solution for classical SB

H. Föllmer (1985-87)

rigorous large deviations over path space interpretation for classical SB





T. Mikami (1990)

P. Dai Pra (1991)

M. Pavon (1991)

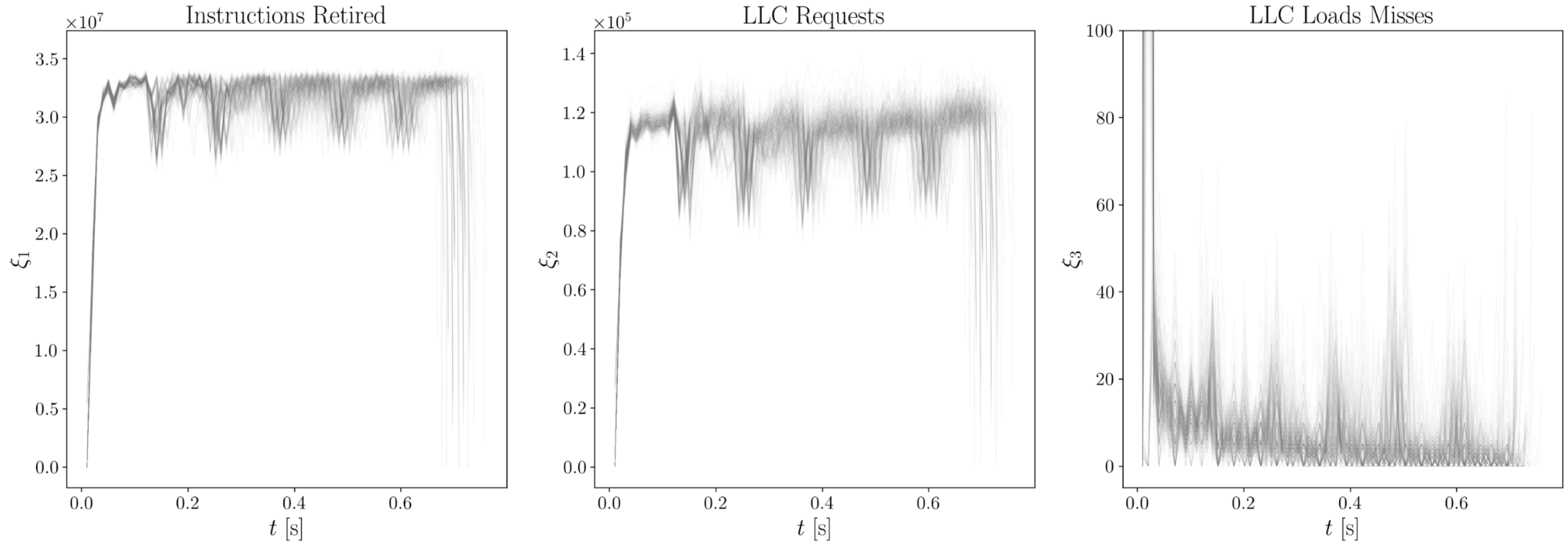
reformulate classical SB as minimum effort stochastic optimal control problem

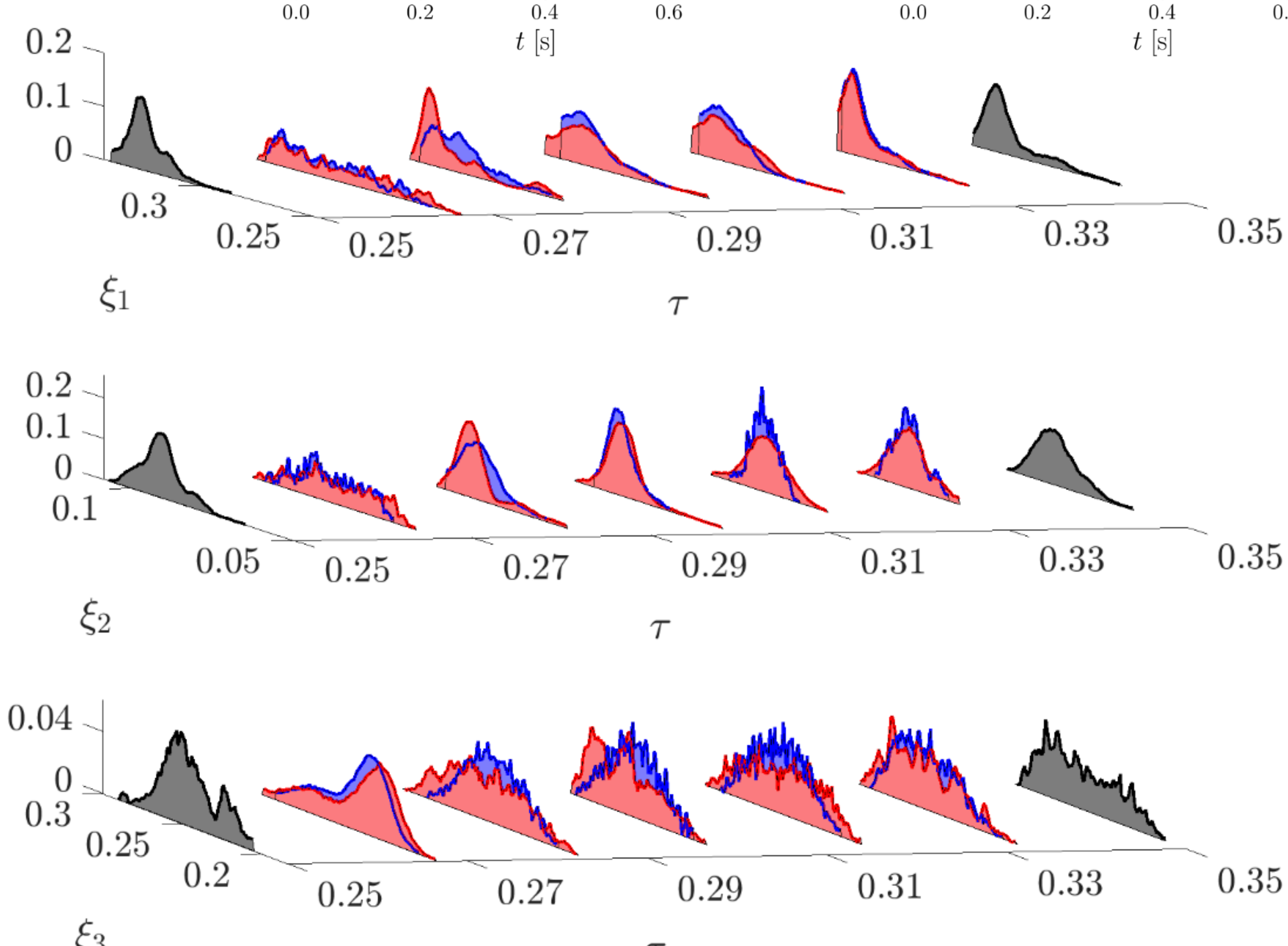
A. Wakolbinger (1991)

A. Blaquière (1991)

SB in the 21st Century: ML Inference

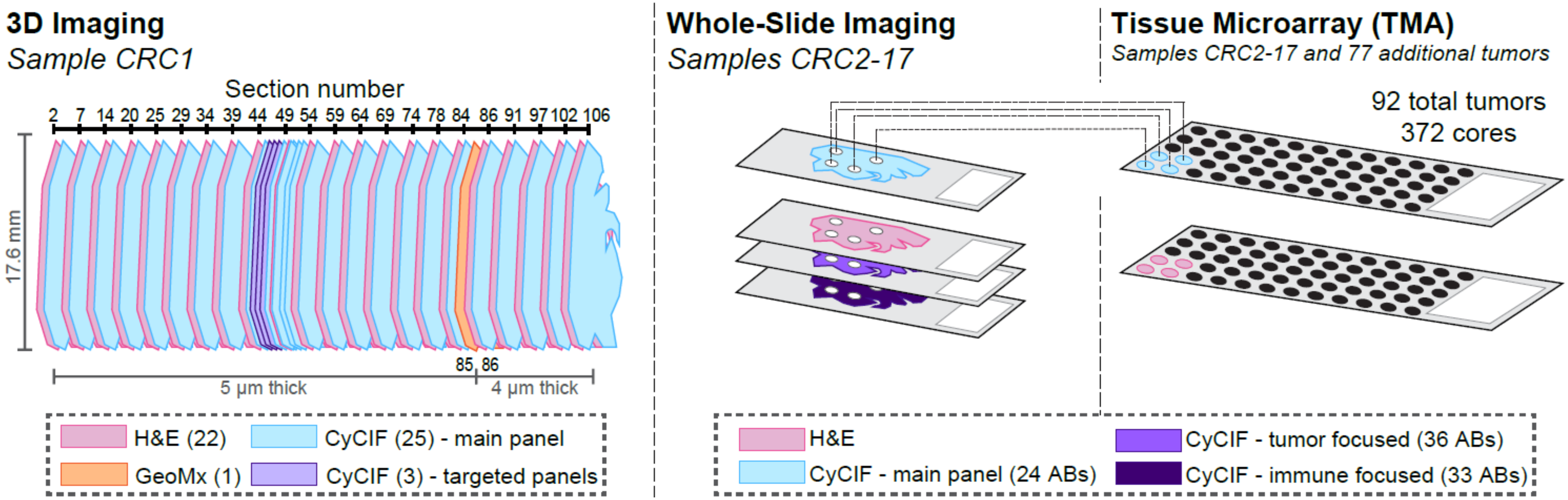
Learn most-likely progression of … multi-core computational resource

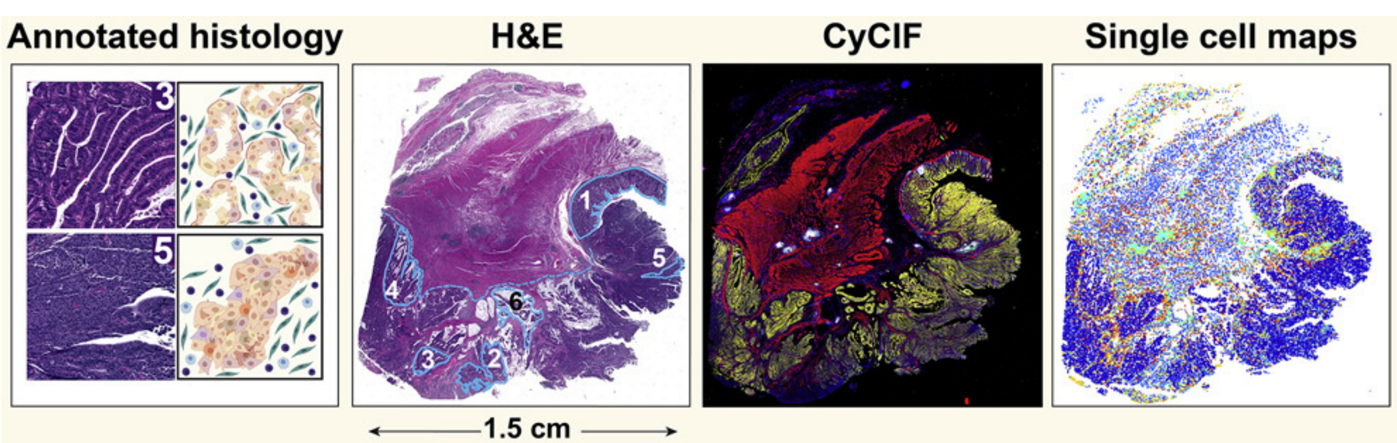


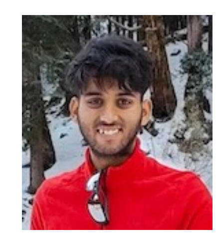


SB in the 21st Century: ML Inference

Learn most-likely progression of … spatial proteomics from tissue slices







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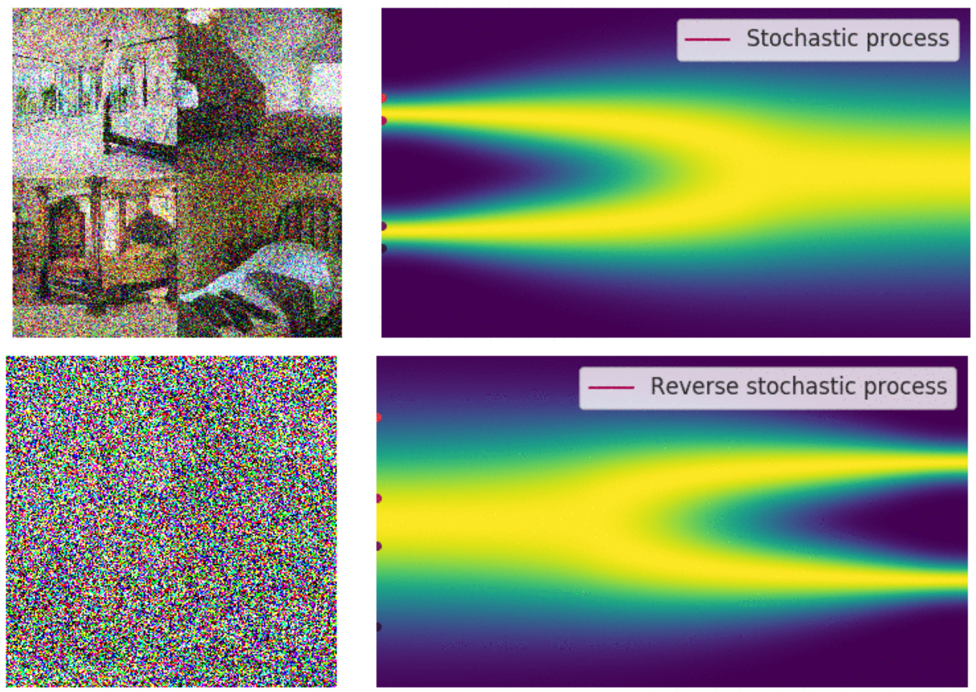
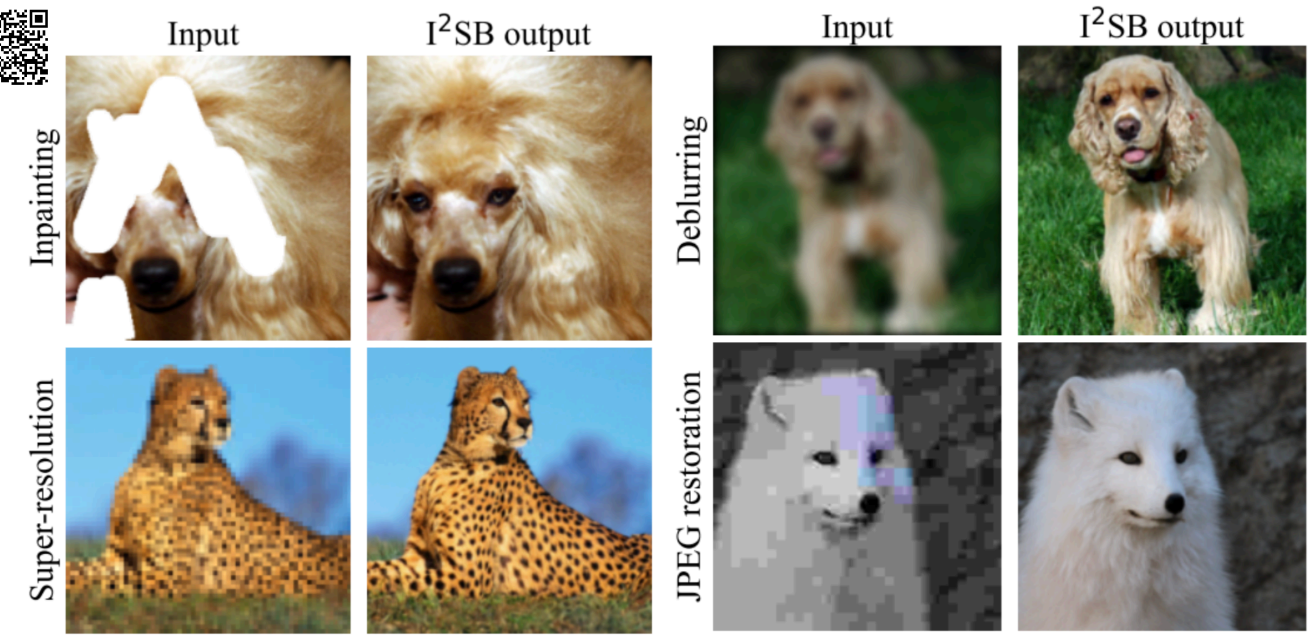
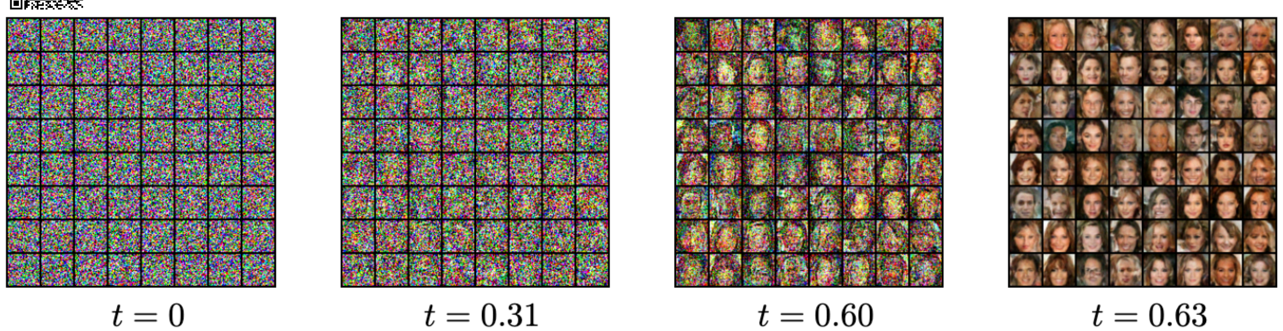
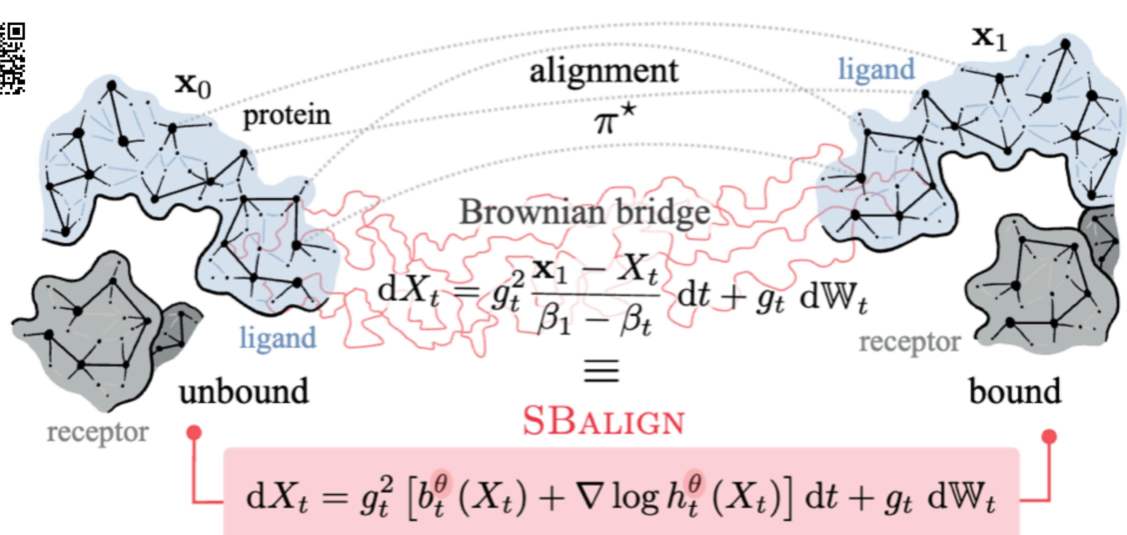
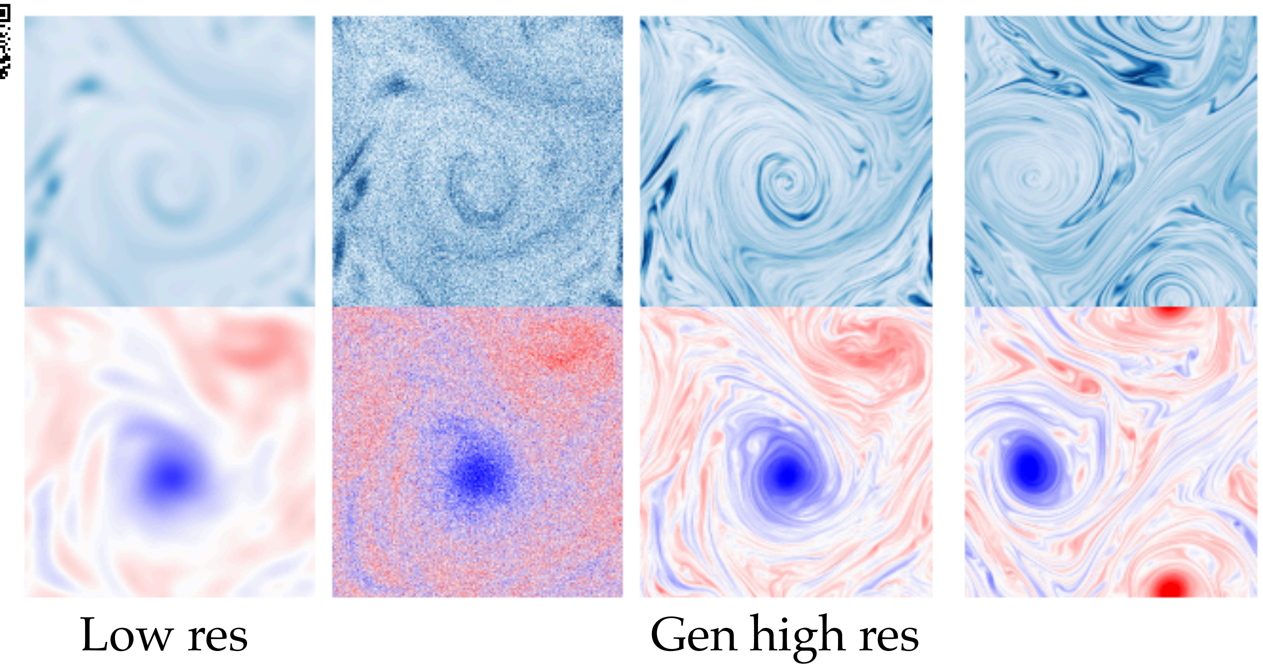
SB in the 21st Century: Generative AI







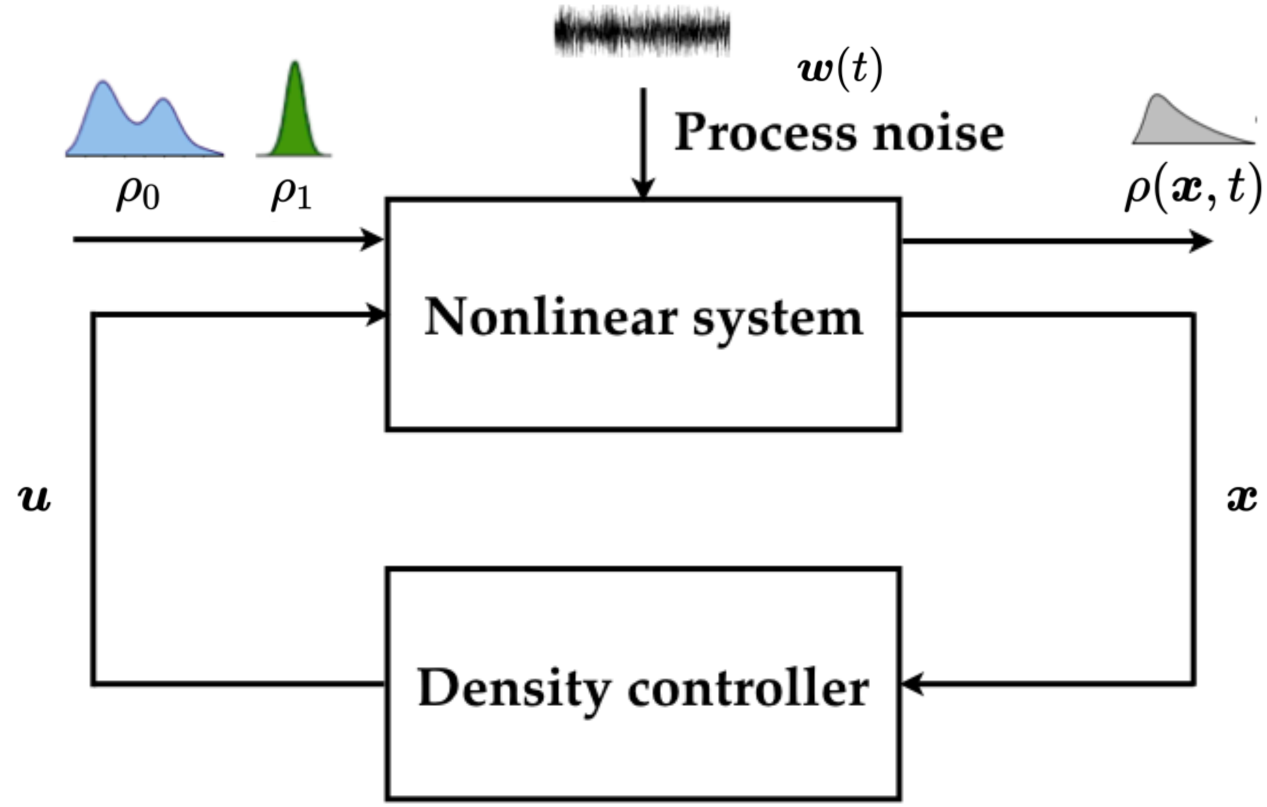




SB as Control Problem

Control of distributions

Allows significant generalizations



SB as Control Problem

Control of distributions

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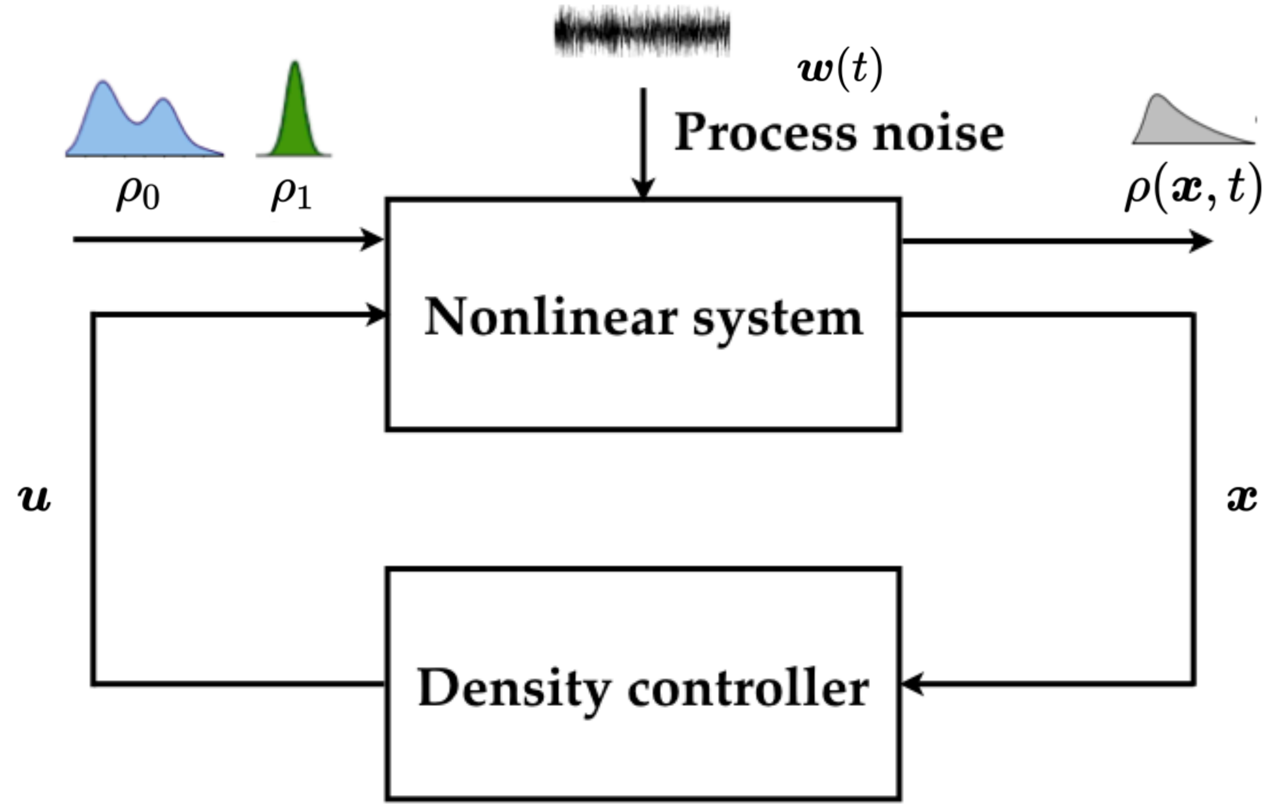


state cost, control cost

$$\underset{(\rho,\boldsymbol{u})\in\mathcal{P}_{01}\times\mathcal{U}}{\arg\inf} \int_{t_0}^{t_1}\int_{\mathbb{R}^n} \left(q(\boldsymbol{x}) + r(\boldsymbol{u})\right)\rho(t,\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}t$$

weighted Laplacian $\sum_{i,j=1}^{n} \frac{\partial^2}{\partial_{x_i}\partial_{x_j}}((\boldsymbol{\Sigma})_{ij}\rho)$

$$\frac{\partial\rho}{\partial t} + \nabla_{\boldsymbol{x}}\cdot(\rho \boldsymbol{f}(t,\boldsymbol{x},\boldsymbol{u})) = \Delta_{\boldsymbol{\Sigma}(t,\boldsymbol{x},\boldsymbol{u})}\rho$$

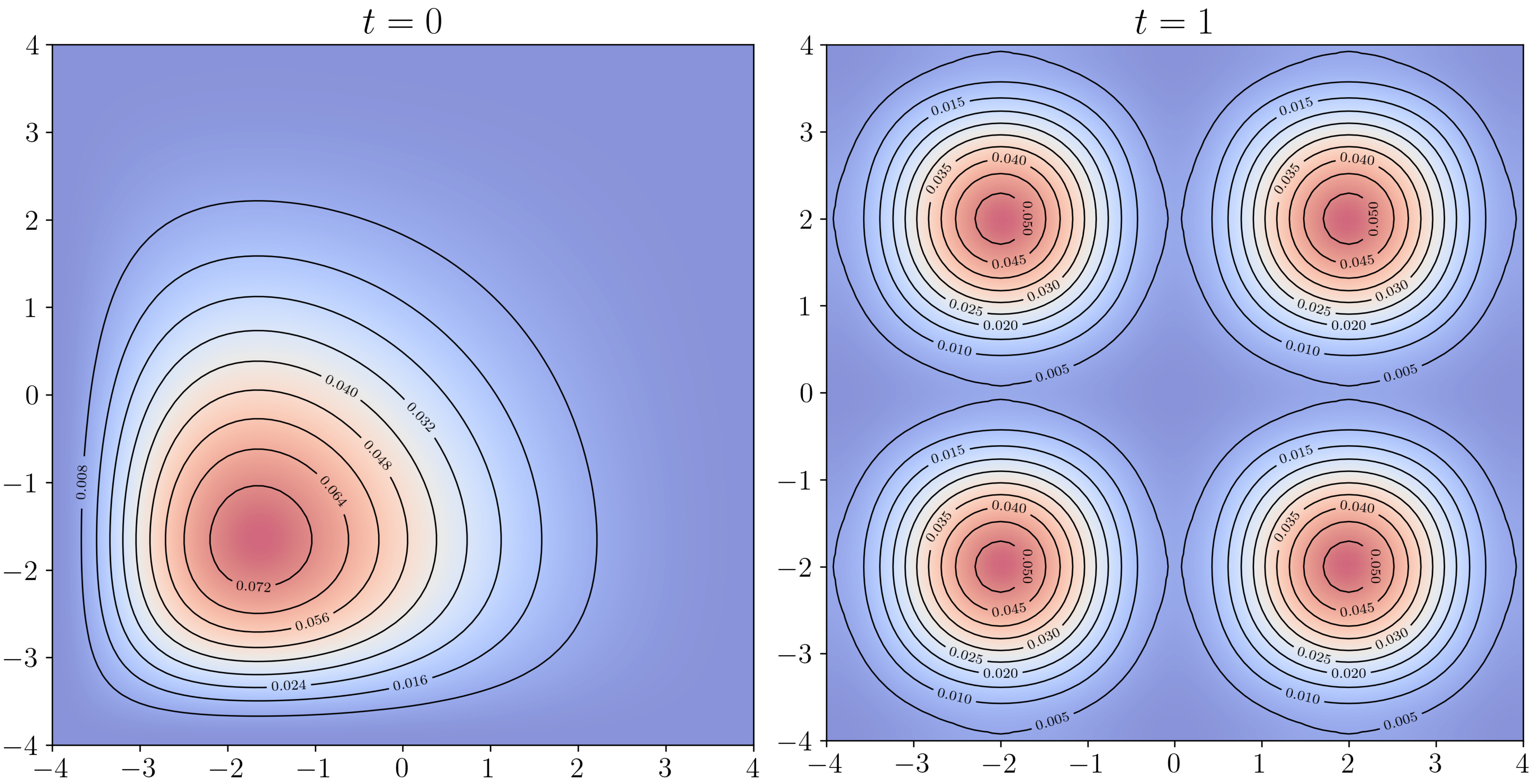
$$\rho(t_0,\boldsymbol{x}) = \rho_0(\boldsymbol{x}), \quad \rho(t_1,\boldsymbol{x}) = \rho_1(\boldsymbol{x}).$$

PDF dynamics for the controlled Itô SDE

prior drift, prior diffusion, diffusion tensor

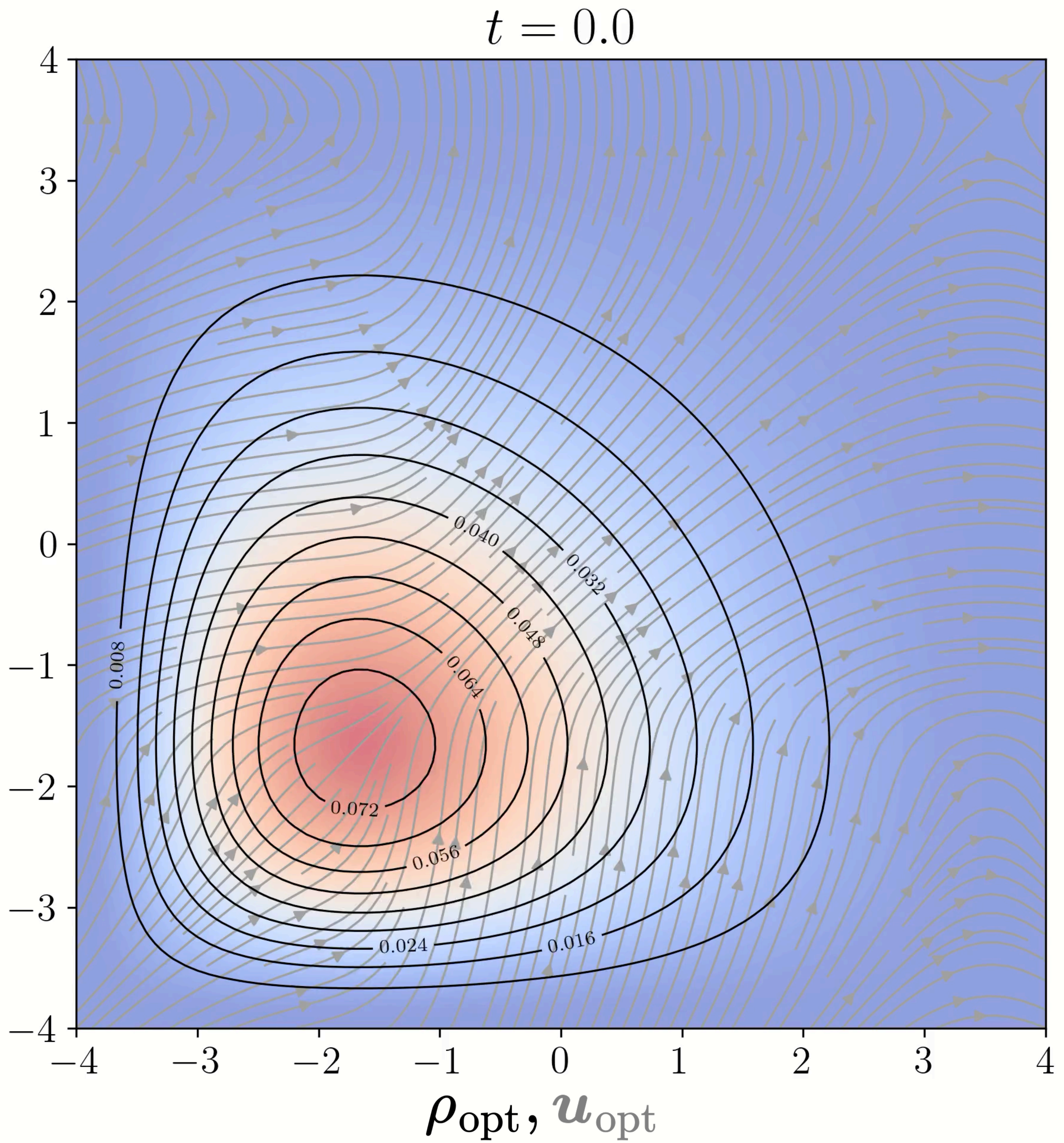
$$\mathrm{d}\boldsymbol{x} = \boldsymbol{f}(t,\boldsymbol{x},\boldsymbol{u})\mathrm{d}t + \boldsymbol{\sigma}(t,\boldsymbol{x},\boldsymbol{u})\mathrm{d}\boldsymbol{w}, \quad \boldsymbol{\Sigma} := \boldsymbol{\sigma}\boldsymbol{\sigma}^\top$$

SB as a Control Problem: Example



ρ_0 ρ_1

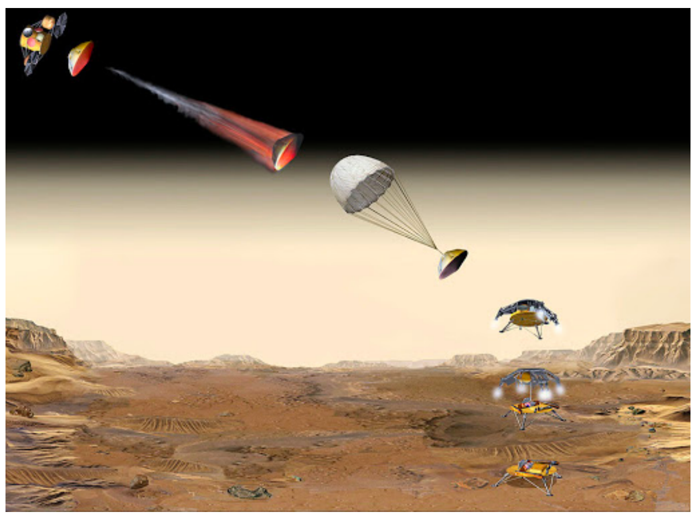
SB as a Control Problem: Example

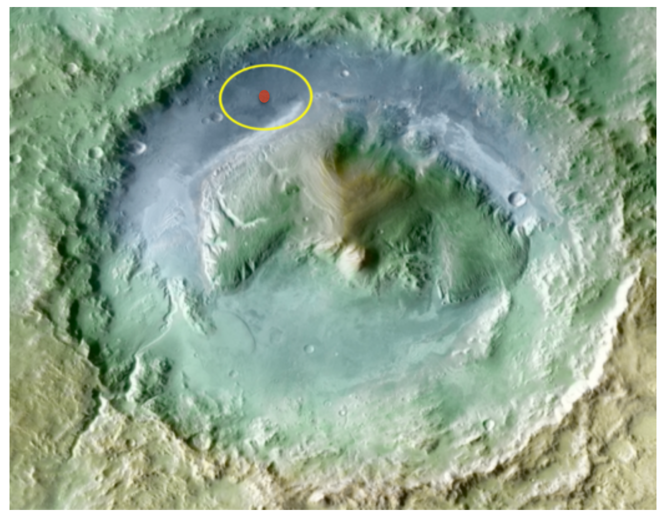


SB as Control Problem: Motivation

Distribution ~ Probability

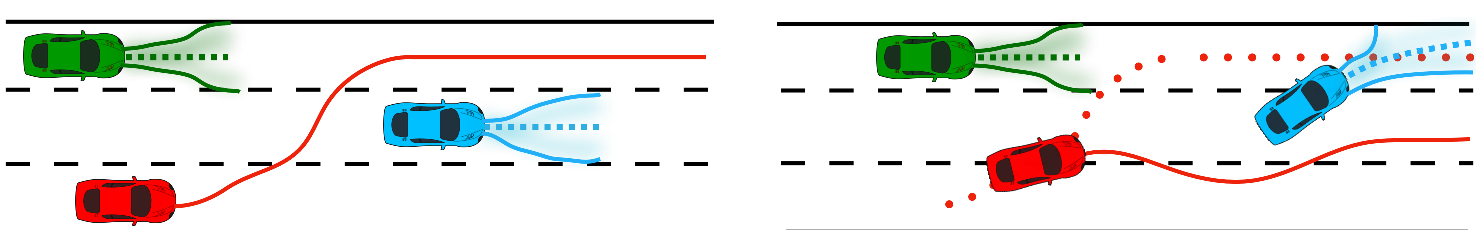
Spacecraft landing with desired statistical accuracy





Gale Crater (4.49S, 137.42E)

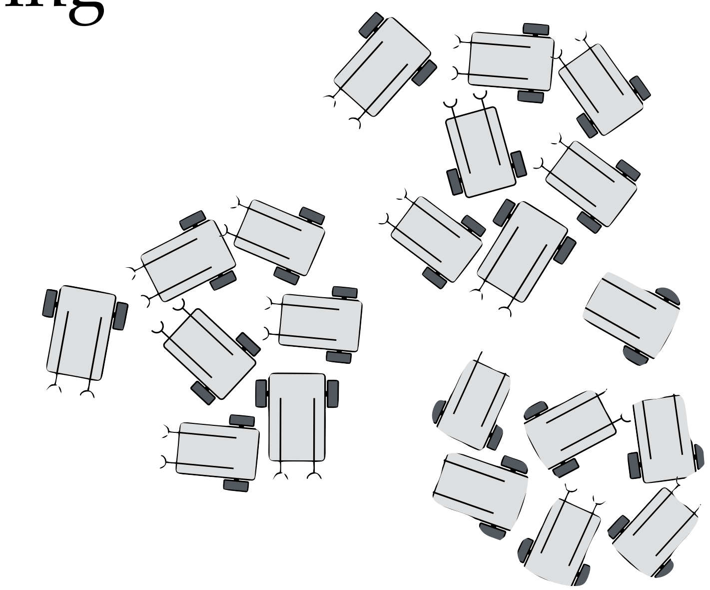
Risk management for automated driving in multi-lane highways



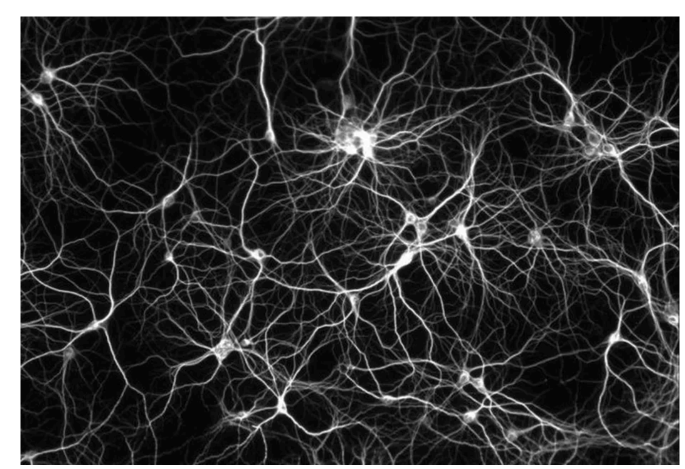
Control of uncertainties

Distribution ~ Population

Dynamic shaping of swarms



Feedback sync. and desync. of neuronal population



Control of ensemble

Rest of this Talk: Generalized SB

#1. General controlled dynamics

#2. Hard sample path constraints

#3. Additive state cost

Generalization #1: General Controlled Dynamics

$$\inf_{(\rho,\boldsymbol{u})\in\mathcal{P}_{01}\times\mathcal{U}} \int_{\mathbb{R}^n}\int_0^1 \left(\frac{1}{2}\|\boldsymbol{u}(t,\boldsymbol{x}_t^u)\|_2^2 + q(t,\boldsymbol{x}_t^u)\right)\rho(t,\boldsymbol{x}_t^u)\mathrm{d}t\,\mathrm{d}\boldsymbol{x}_t^u$$

subject to

$$\frac{\partial\rho}{\partial t} + \nabla\cdot((\boldsymbol{f}+\boldsymbol{g}\,\boldsymbol{u})\rho) = \Delta_{\boldsymbol{\Sigma}}\rho$$

$$\rho(t=0,\boldsymbol{x}_0^u) = \rho_0, \quad \rho(t=1,\boldsymbol{x}_1^u) = \rho_1$$

This is control-affine SB

Diffusion tensor: $\boldsymbol{\Sigma} := \boldsymbol{\sigma}\boldsymbol{\sigma}^\top \succeq \boldsymbol{0}$

Zero noise limit $\Rightarrow \Delta_{\boldsymbol{\Sigma}}\rho = 0 \Rightarrow$ control-affine optimal transport

Necessary Conditions for Optimality

Coupled nonlinear PDEs + linear boundary conditions

Primal PDE:

$$\frac{\partial \rho_{\rm opt}^{\boldsymbol{u}}}{\partial t} + \nabla_{\boldsymbol{x}} \cdot \left(\rho_{\rm opt}^{\boldsymbol{u}} \left(\boldsymbol{f} + \boldsymbol{g}\boldsymbol{g}^{\top} \nabla_{\boldsymbol{x}} S\right)\right) = \frac{1}{2} \Delta_{\boldsymbol{\Sigma}}\, \rho_{\rm opt}^{\boldsymbol{u}}$$

Dual PDE:

$$\frac{\partial S}{\partial t} + \langle \nabla_{\boldsymbol{x}} S, \boldsymbol{f} \rangle + \frac{1}{2} \langle \nabla_{\boldsymbol{x}} S, \boldsymbol{g}\boldsymbol{g}^{\top} \nabla_{\boldsymbol{x}} S \rangle + \frac{1}{2} \langle \boldsymbol{\Sigma}, \mathrm{Hess}_{\boldsymbol{x}} S \rangle = q$$

Primal boundary conditions:

$$\rho_{\rm opt}^{\boldsymbol{u}}\left(t = t_0, \cdot\right) = \rho_0(\cdot),\; \rho_{\rm opt}^{\boldsymbol{u}}\left(t = t_1, \cdot\right) = \rho_1(\cdot)$$

Optimal control: $\quad \boldsymbol{u}_{\rm opt} = \boldsymbol{g}^{\top} \nabla_{\boldsymbol{x}} S$

Feedback Synthesis via Schrödinger Factors $(\hat{\varphi}, \varphi)$

Hopf-Cole a.k.a. Fleming's logarithmic transform for some scaling constant $\lambda > 0$:

$$\left(\rho_{\rm opt}^{\boldsymbol{u}}, S\right) \mapsto \left(\widehat{\varphi}, \varphi\right) := \left(\rho_{\rm opt}^{\boldsymbol{u}} \exp\left(-S/\lambda\right), \exp\left(S/\lambda\right)\right)$$

Boundary-coupled system of PDEs for the factors:

$$\frac{\partial \widehat{\varphi}}{\partial t} + \nabla_{\boldsymbol{x}} \cdot \left(\widehat{\varphi}\left(\boldsymbol{f} + \boldsymbol{f}_{\varphi}\right)\right) - \frac{1}{2}\Delta_{\boldsymbol{\Sigma}}\,\widehat{\varphi} + \left(\frac{q}{\lambda} + q_{\varphi}\right)\widehat{\varphi} = 0$$

$$\frac{\partial \varphi}{\partial t} + \langle \nabla_{\boldsymbol{x}}\varphi, \boldsymbol{f} + \boldsymbol{f}_{\varphi}\rangle + \frac{1}{2}\langle \boldsymbol{\Sigma}, \mathrm{Hess}_{\boldsymbol{x}}\varphi\rangle - \left(\frac{q}{\lambda} + q_{\varphi}\right)\varphi = 0$$

$\boldsymbol{f}_{\varphi}$: $(\lambda \boldsymbol{g}\boldsymbol{g}^{\top} - \boldsymbol{\Sigma})\nabla_{\boldsymbol{x}} \log \varphi$

q_{φ}: $\frac{1}{2}(\nabla_{\boldsymbol{x}} \log \varphi)^{\top}(\lambda \boldsymbol{g}\boldsymbol{g}^{\top} - \boldsymbol{\Sigma})\nabla_{\boldsymbol{x}} \log \varphi$

$$\widehat{\varphi}(t_0, \cdot)\varphi(t_0, \cdot) = \rho_0(\cdot),\ \widehat{\varphi}(t_1, \cdot)\varphi(t_1, \cdot) = \rho_1(\cdot)$$

PDEs become linear + equation-level decoupled if $\boldsymbol{g}\boldsymbol{g}^{\top} \propto \boldsymbol{\Sigma}$

What Exactly are the Schrödinger Factors?

Consider classical SB: $\boldsymbol{f} = \mathbf{0}, \ \boldsymbol{g} = \boldsymbol{\Sigma} = \boldsymbol{I}$

Classical: $\rho^{\text{opt}}(\boldsymbol{x}, t) = \varphi(\boldsymbol{x}, t)\widehat{\varphi}(\boldsymbol{x}, t)$

$$\left(\frac{\partial}{\partial t} + \frac{1}{2}\Delta - q\right)\varphi = 0$$ [Backward reaction-diffusion PDE]

$$\left(\frac{\partial}{\partial t} - \frac{1}{2}\Delta + q\right)\widehat{\varphi} = 0$$ [Forward reaction-diffusion PDE]

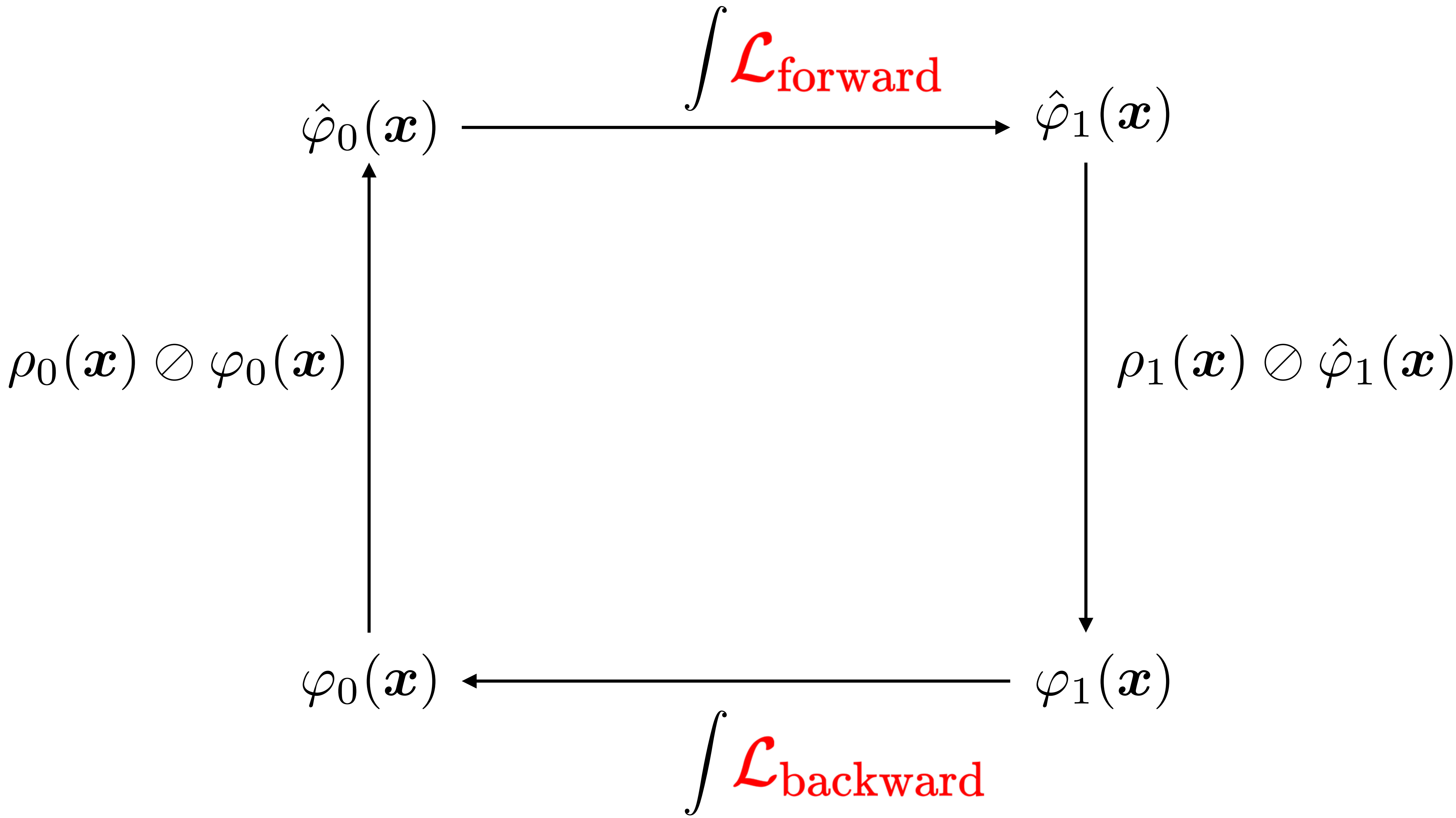
Quantum: $\rho^{\text{opt}}(\boldsymbol{x}, t) = \Psi(\boldsymbol{x}, t)\widehat{\Psi}(\boldsymbol{x}, t)$ [Born's relation]

wave function

$$\left(\sqrt{-1}\frac{\partial}{\partial t} + \frac{1}{2}\Delta - q\right)\Psi = 0$$ [Schrödinger PDE]

$$\left(-\sqrt{-1}\frac{\partial}{\partial t} - \frac{1}{2}\Delta + q\right)\widehat{\Psi} = 0$$ [Adjoint Schrödinger PDE]

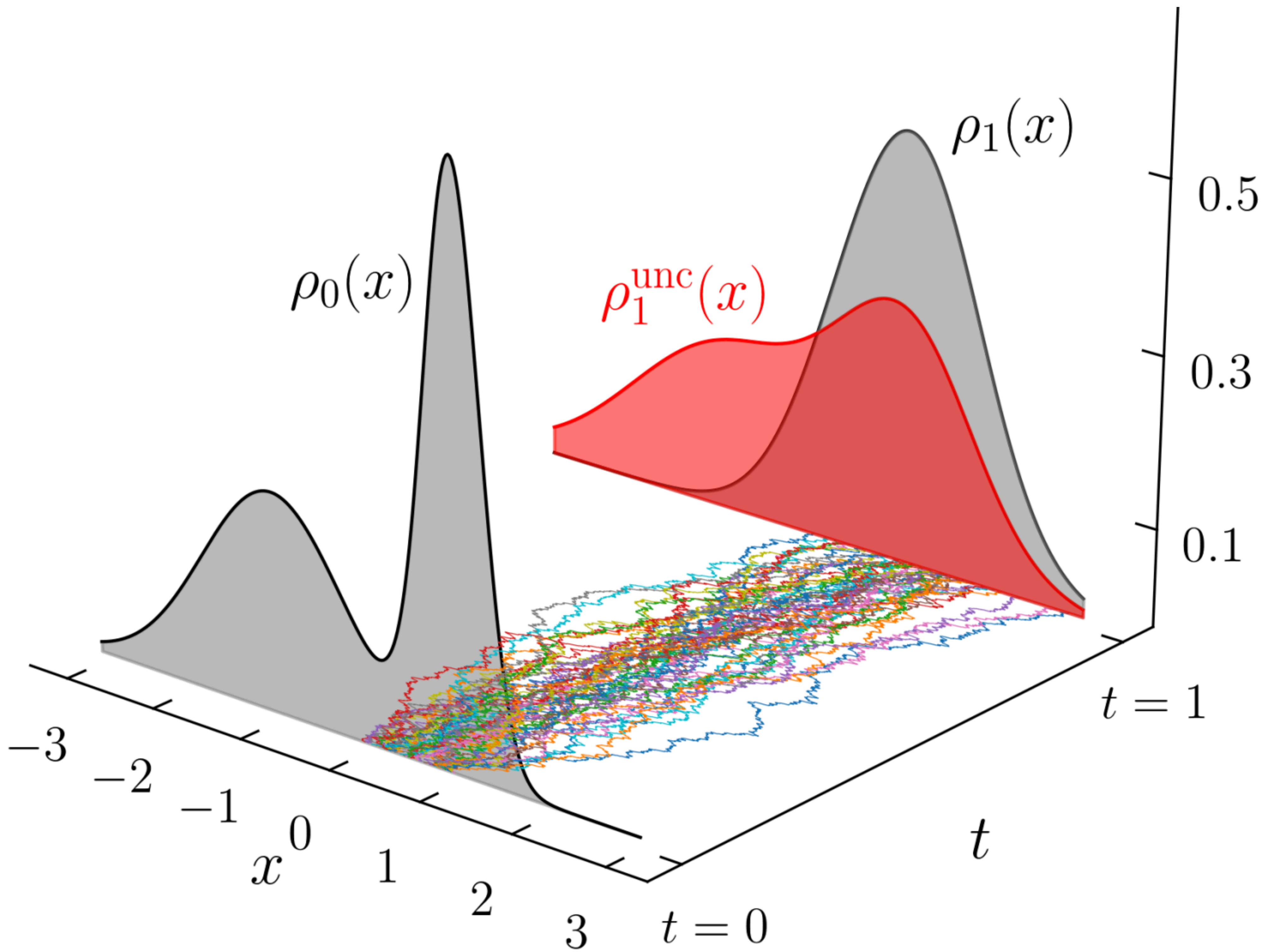
Fixed Point Recursion Over Pair $(\varphi_1, \hat{\varphi}_0)$



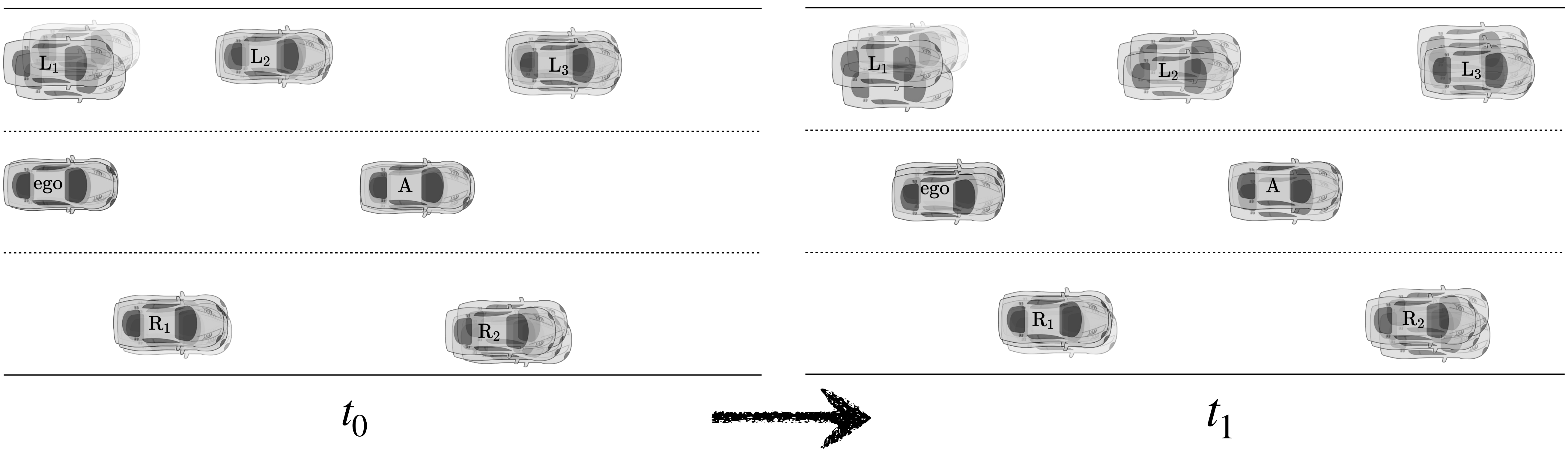
This recursion is contractive in the Hilbert's projective metric!!

Numerical Example: Classical SB

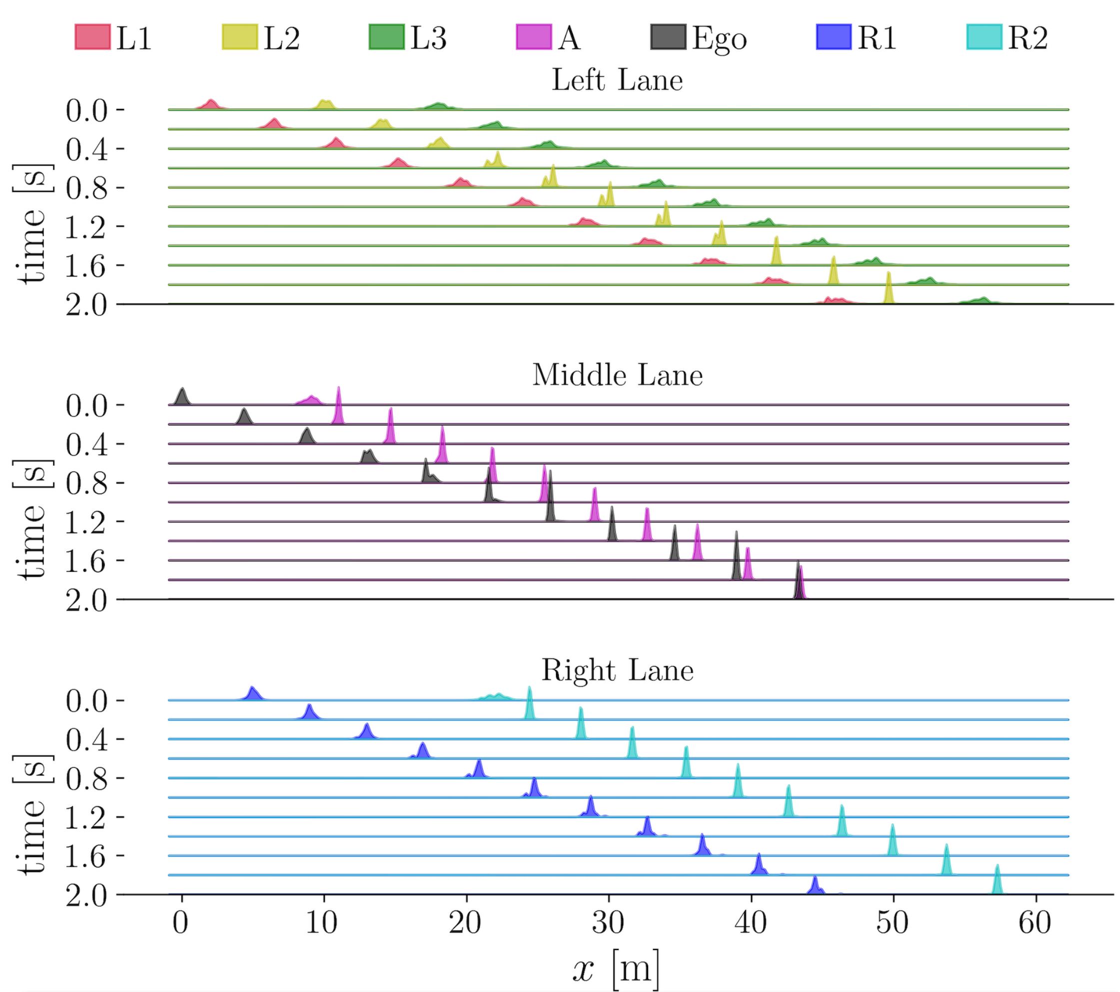
$$\boldsymbol{f} = \boldsymbol{0},\ \boldsymbol{g} = \boldsymbol{\Sigma} = \boldsymbol{I}, q = 0$$



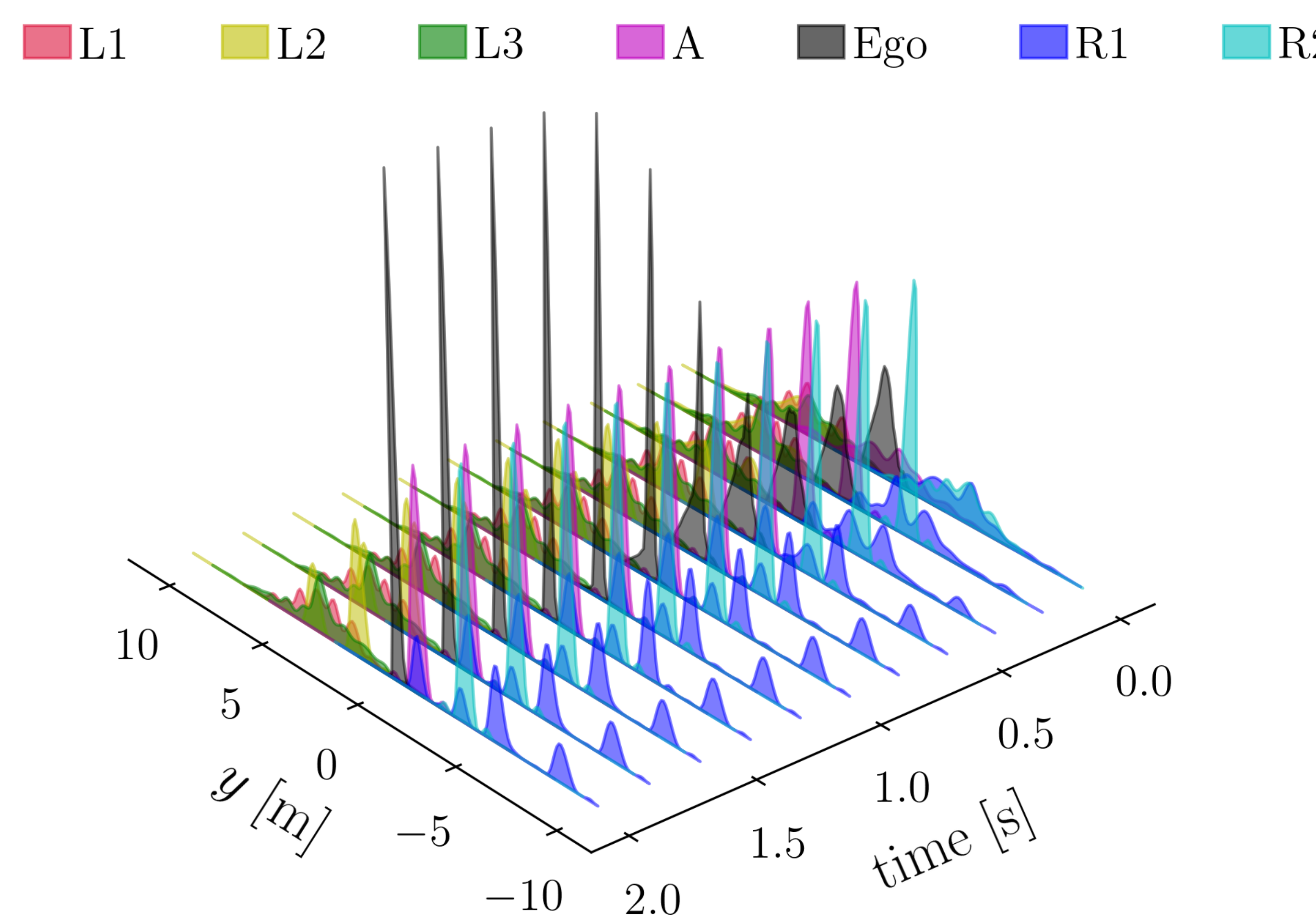
Application: Multi-lane Automated Driving



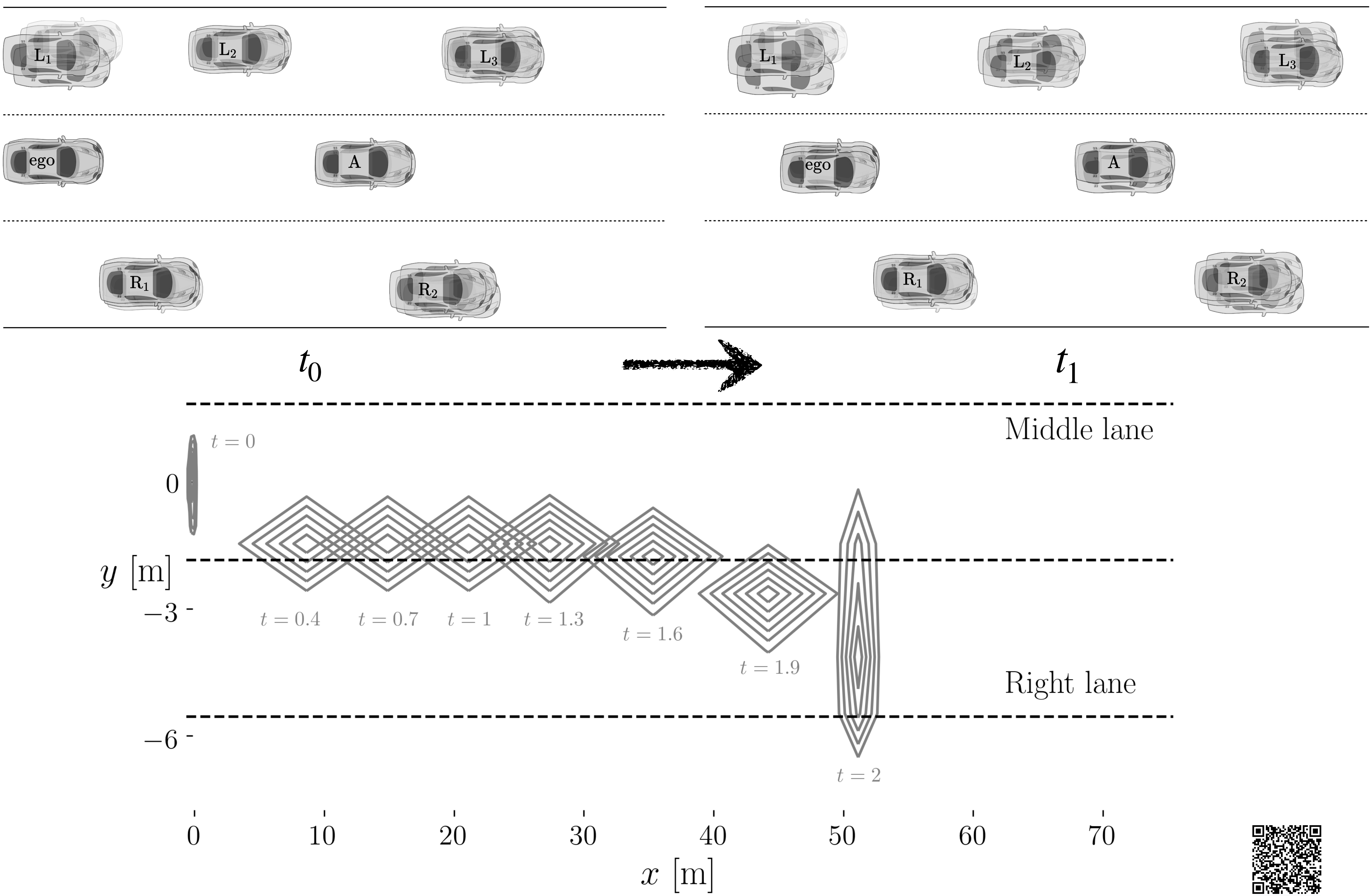
x marginals



y marginals



Application: Multi-lane Automated Driving

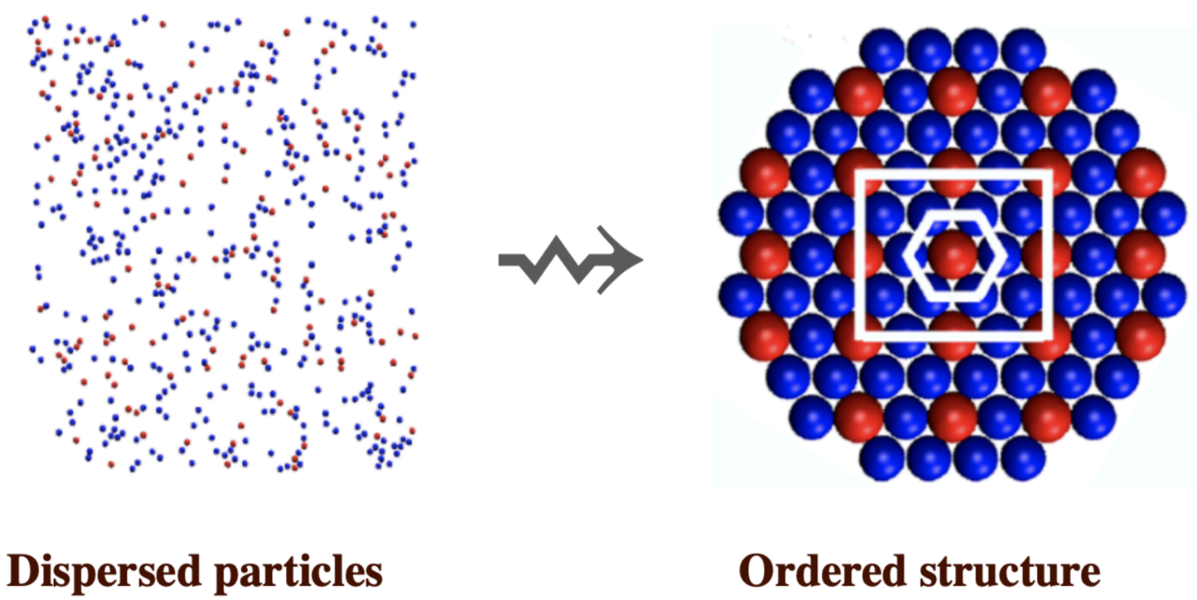


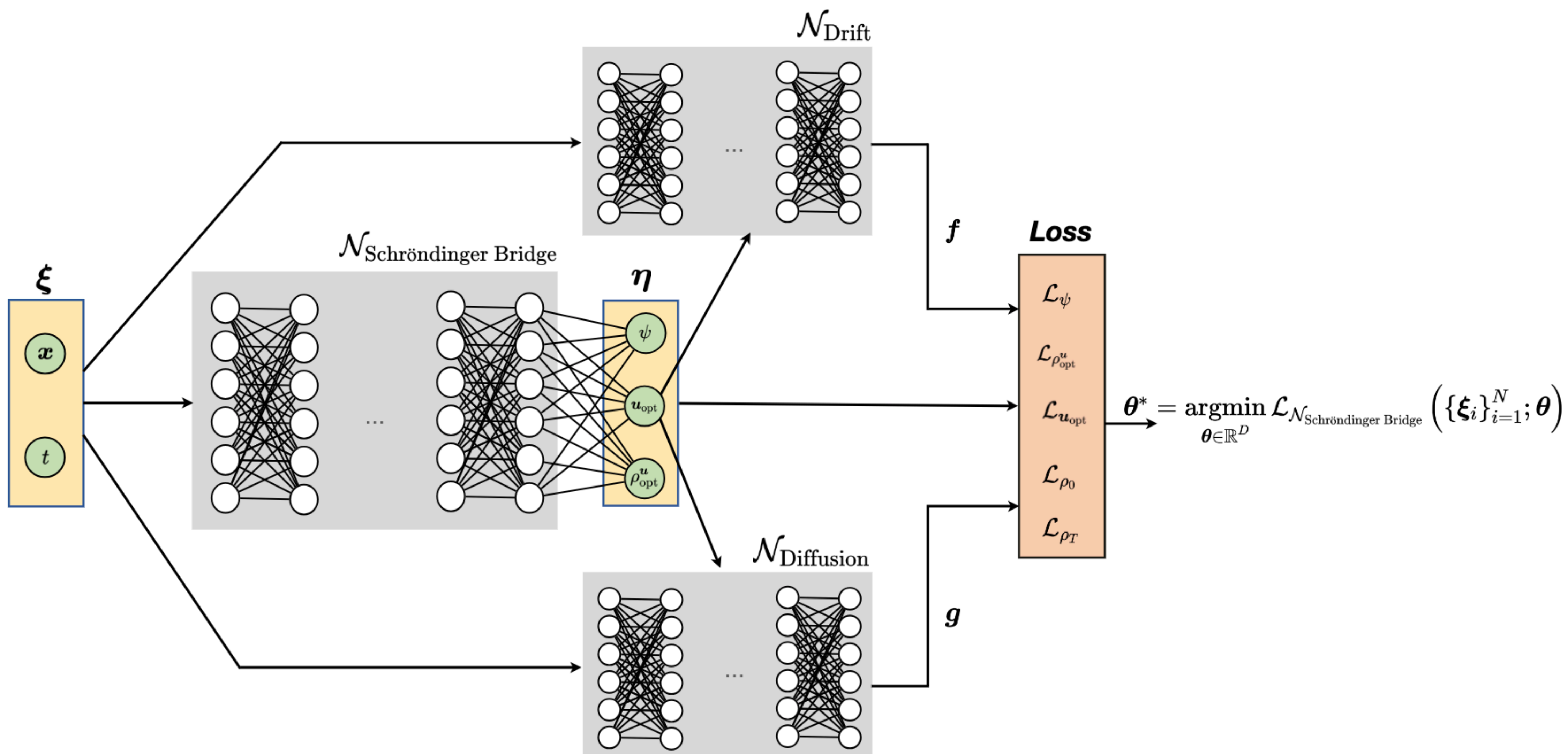
Application: Controlled Colloidal Self-assembly

control non-affine SB

Conditions for optimality:
system of $m + 2$ coupled nonlinear PDEs

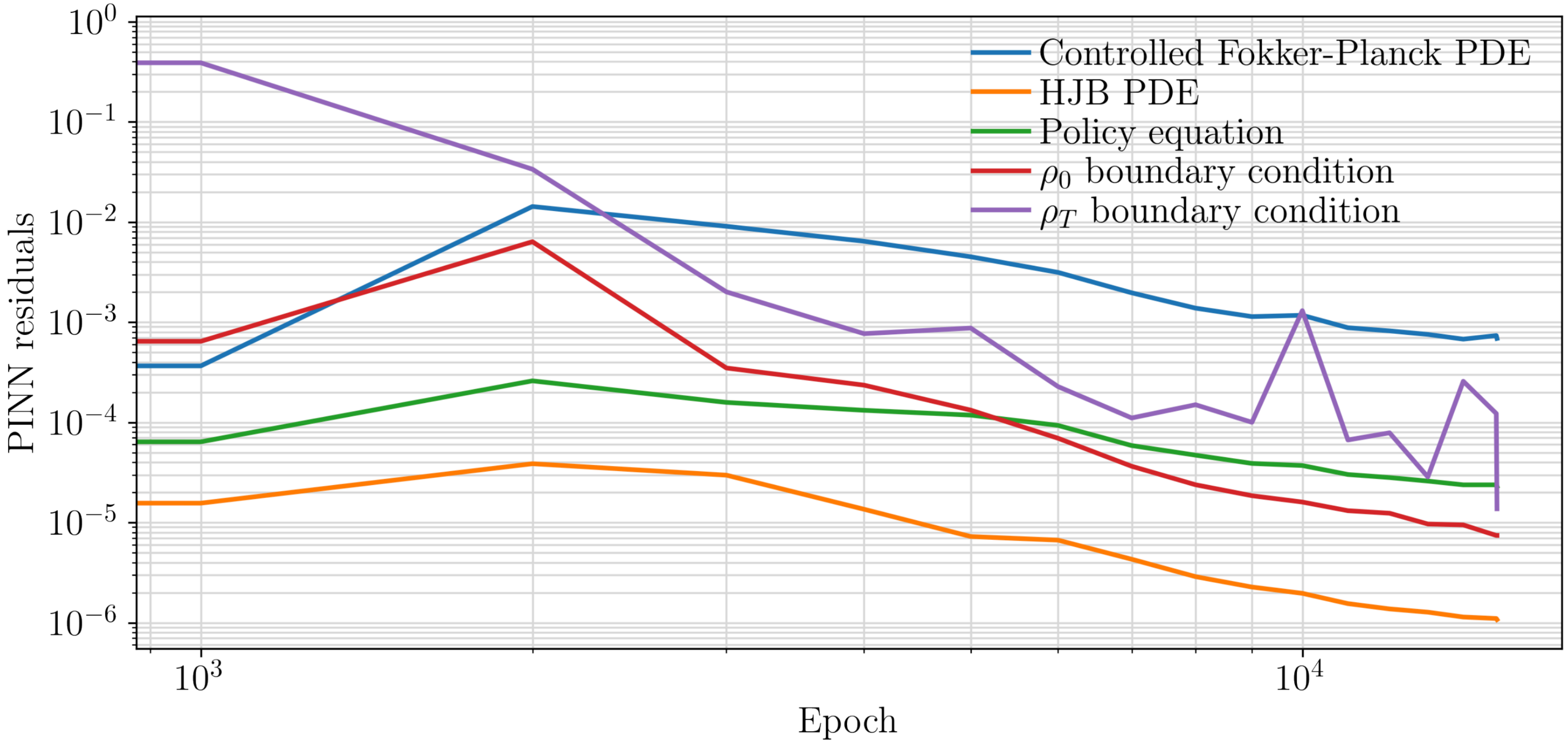
Data-driven drift and diffusion coeff.





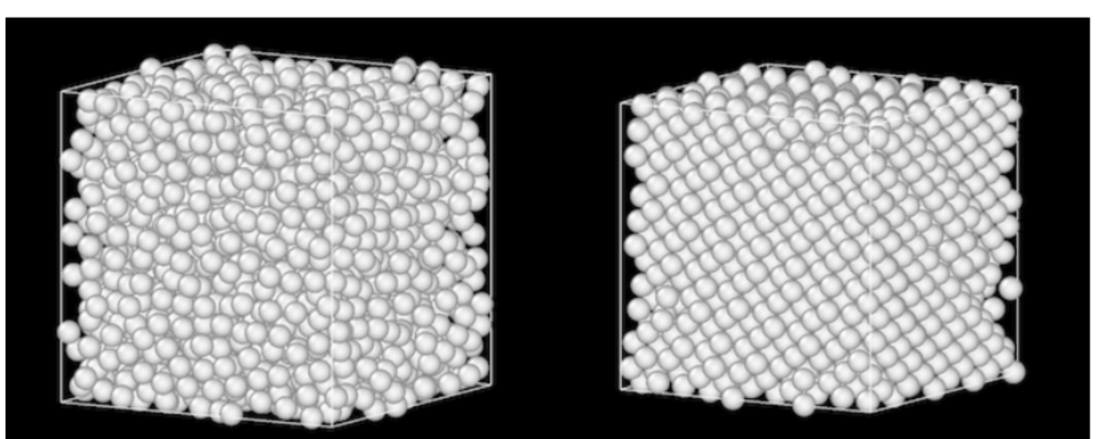
Application: Controlled Colloidal Self-assembly

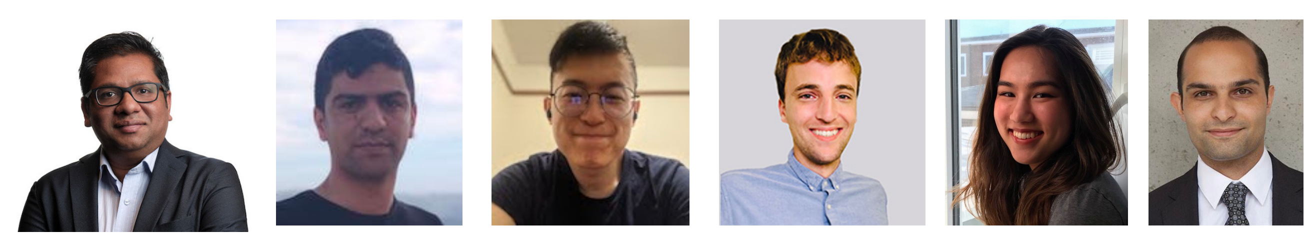
Benchmark controlled self-assembly system: [Y Xue, et al, *IEEE Trans. Control Sys. Technology*, 2014]



2024 O. Hugo Schuck Best Application Paper Award

BCC structure synthesis via control non-affine SB:





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Generalization #2: Hard Sample Path Constraints

Main idea: path constraints $\sim$ reflected Itô SDEs

modify the controlled sample path dynamics to

$$\mathrm{d}\boldsymbol{x}_t^{\boldsymbol{u}} = \{\boldsymbol{f}(t,\boldsymbol{x}_t^{\boldsymbol{u}}) + \boldsymbol{B}(t)\boldsymbol{u}(t,\boldsymbol{x}_t^{\boldsymbol{u}})\}\mathrm{d}t + \sqrt{2\theta}\boldsymbol{G}(t)\mathrm{d}\boldsymbol{w}_t + \boldsymbol{n}(\boldsymbol{x}_t^{\boldsymbol{u}})\mathrm{d}\gamma_t$$

$\boldsymbol{x}_t^{\boldsymbol{u}} \in \overline{\mathcal{X}} := \mathcal{X} \cup \partial\mathcal{X}$, closure of connected smooth $\mathcal{X}$

$\boldsymbol{n}$ is inward unit normal to the boundary $\partial\mathcal{X}$

γ_t is minimal local time stochastic process

Reflected SB: Schrödinger Factor Recursion

$$\int \text{with b.c. } \langle \boldsymbol{f}\hat{\varphi} - \theta\nabla\hat{\varphi}, \boldsymbol{n}\rangle\big|_{\partial\mathcal{X}} = 0$$

$$\hat{\varphi}_0(\boldsymbol{x}) \longrightarrow \hat{\varphi}_1(\boldsymbol{x})$$

$$\rho_1(\boldsymbol{x}) \oslash \hat{\varphi}_1(\boldsymbol{x})$$

$$\varphi_0(\boldsymbol{x}) \longleftarrow \varphi_1(\boldsymbol{x})$$

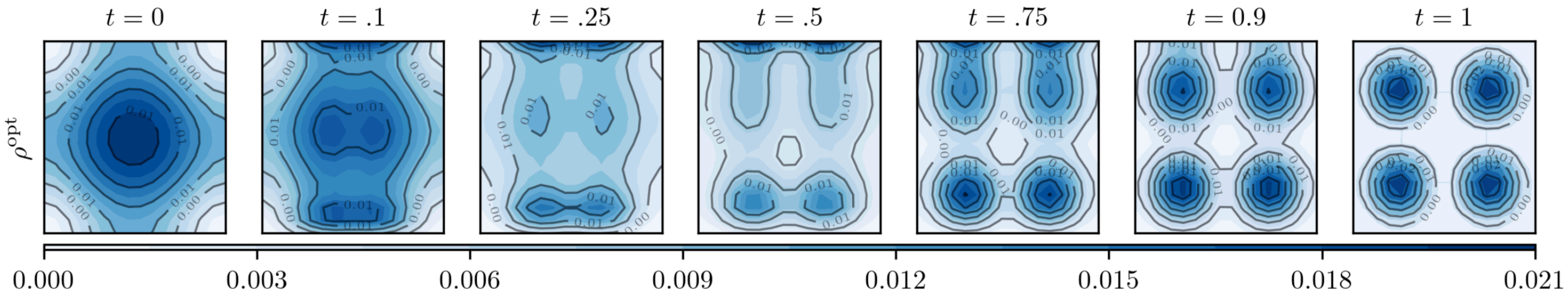
$$\int \text{with b.c. } \langle \nabla\varphi, \boldsymbol{n}\rangle\big|_{\partial\mathcal{X}} = 0$$

$$\rho_0(\boldsymbol{x}) \oslash \varphi_0(\boldsymbol{x})$$

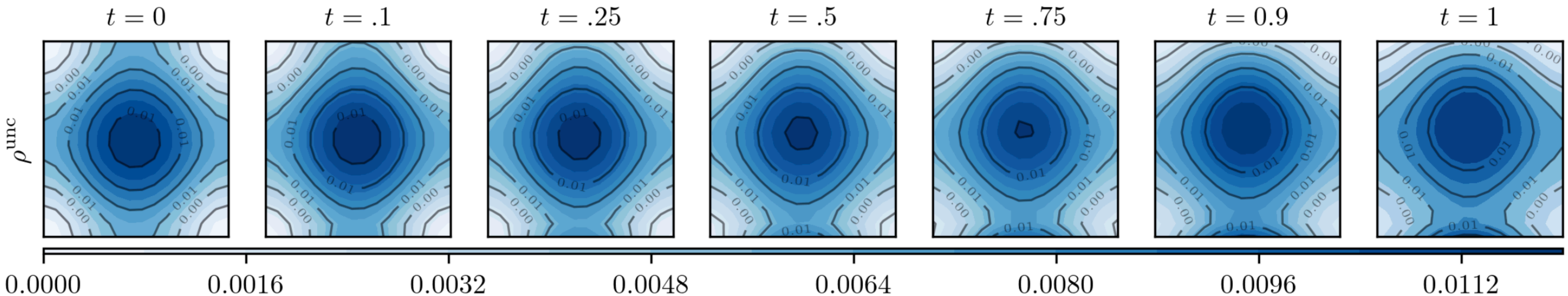
Reflected Bridge: Numerics with ∇V Drift

$$V(x_1, x_2) = (x_1^2 + x_2^3)/5, \quad \overline{\mathcal{X}} = [-4, 4]^2$$

Optimal controlled state PDFs:



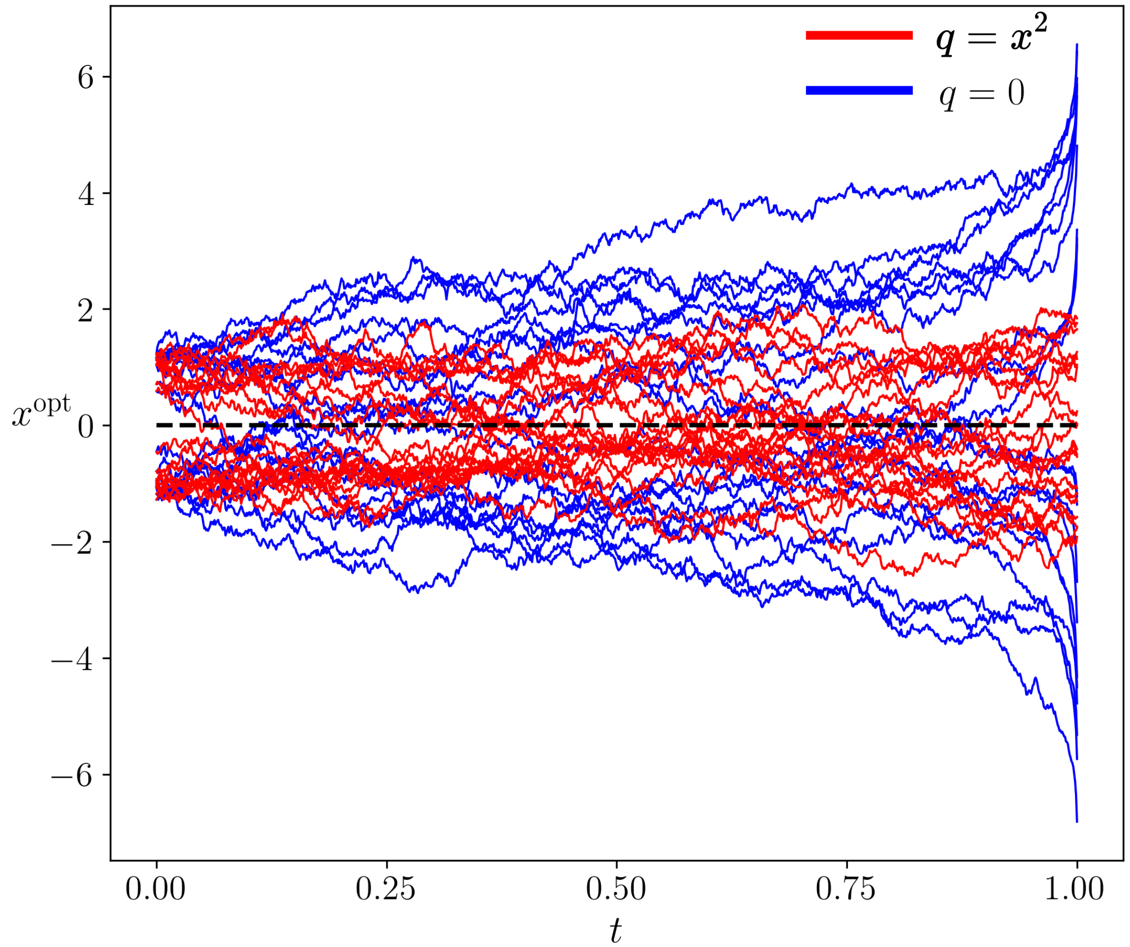
Uncontrolled state PDFs:



Generalization #3: Additive State Cost ($q \neq 0$)

Question. Where does state cost come from?

Answer 1. From extra regularization (e.g., classical LQ optimal control)



Answer 2. Problem reformulation (push dynamical nonlinearity to Lagrangian)

Probabilistic Lambert Problem: Connections with Optimal Mass Transport, Schrödinger Bridge and Reaction-Diffusion PDEs*

Alexis M.H. Teter†, Iman Nodozi‡, and Abhishek Halder§

SB with Quadratic State Cost: $q(\boldsymbol{x}) = \boldsymbol{x}^\top Q \boldsymbol{x}, Q \geq 0$

Solution: $\rho^{\text{opt}}(\boldsymbol{x}, t) = \varphi(\boldsymbol{x}, t)\widehat{\varphi}(\boldsymbol{x}, t)$

$$\left(\frac{\partial}{\partial t} + \frac{1}{2}\Delta - q\right)\varphi = 0$$

[Backward reaction-diffusion PDE]

$$\left(\frac{\partial}{\partial t} - \frac{1}{2}\Delta + q\right)\widehat{\varphi} = 0$$

[Forward reaction-diffusion PDE]

SB with Quadratic State Cost: $q(x) = x^\top Q x, Q \geq 0$

We know: $\rho^{\text{opt}}(\boldsymbol{x}, t) = \varphi(\boldsymbol{x}, t)\widehat{\varphi}(\boldsymbol{x}, t)$

$$\left(\frac{\partial}{\partial t} + \frac{1}{2}\Delta - q\right)\varphi = 0$$ [Backward reaction-diffusion PDE]

$$\left(\frac{\partial}{\partial t} - \frac{1}{2}\Delta + q\right)\widehat{\varphi} = 0$$ [Forward reaction-diffusion PDE]

Need kernel/Green's function $\kappa\left(0, \boldsymbol{x}; t, \boldsymbol{y}\right)$

for IVP solutions to use in Schrödinger factor recursion:

$$\frac{\partial \widehat{\varphi}}{\partial t} = \underbrace{\mathcal{L}_{\text{forward}}}_{(\Delta - \boldsymbol{x}^\top \boldsymbol{Q} x)} \widehat{\varphi}, \quad \widehat{\varphi}(t = 0, \boldsymbol{x}) = \widehat{\varphi}_0 \quad \Leftrightarrow \quad \widehat{\varphi}(\boldsymbol{x}, t) = \int_{\mathbb{R}^n} \boxed{\kappa(0, \boldsymbol{x}; t, \boldsymbol{z})} \widehat{\varphi}_0(\boldsymbol{z}) \mathrm{d}\boldsymbol{z}$$

SB with Quadratic State Cost: $q(x) = x^\top Qx, Q \geq 0$

Thm. Eig. decomposition: $\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^\top$

Then, $\widehat{\varphi}(\boldsymbol{x}, t) = \eta(\boldsymbol{y} = \boldsymbol{V}\boldsymbol{x}, t)$ where $\eta(\boldsymbol{y}, t) = \int_{\mathbb{R}^n} \kappa(0, \boldsymbol{y}; t, \boldsymbol{z})\eta_0(\boldsymbol{z})\mathrm{d}\boldsymbol{z}$

and

$$\kappa(0, \boldsymbol{y}; t, \boldsymbol{z}) = \frac{(\det(\boldsymbol{D}))^{1/4}}{\sqrt{(2\pi)^n \det\left(\sinh(2t\sqrt{\boldsymbol{D}})\right)}} \exp\left(-\frac{1}{2}(\boldsymbol{y} \quad \boldsymbol{z})\boldsymbol{M}\begin{pmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{pmatrix}\right)$$

$$\boldsymbol{M} := \begin{bmatrix}\boldsymbol{D}^{1/4} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{D}^{1/4}\end{bmatrix}\boldsymbol{M}_1\boldsymbol{M}_2\begin{bmatrix}\boldsymbol{D}^{1/4} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{D}^{1/4}\end{bmatrix}, \quad \boldsymbol{M}_1 := \begin{bmatrix}\cosh\left(2t\sqrt{\boldsymbol{D}}\right) & -\boldsymbol{I}_n\\ -\boldsymbol{I}_n & \cosh\left(2t\sqrt{\boldsymbol{D}}\right)\end{bmatrix}, \quad \boldsymbol{M}_2 := \begin{bmatrix}\operatorname{csch}\left(2t\sqrt{\boldsymbol{D}}\right) & \boldsymbol{0}\\ \boldsymbol{0} & \operatorname{csch}\left(2t\sqrt{\boldsymbol{D}}\right)\end{bmatrix}$$

$$\eta_0(\boldsymbol{y}) = \widehat{\varphi}_0(\boldsymbol{V}^\top\boldsymbol{x})$$

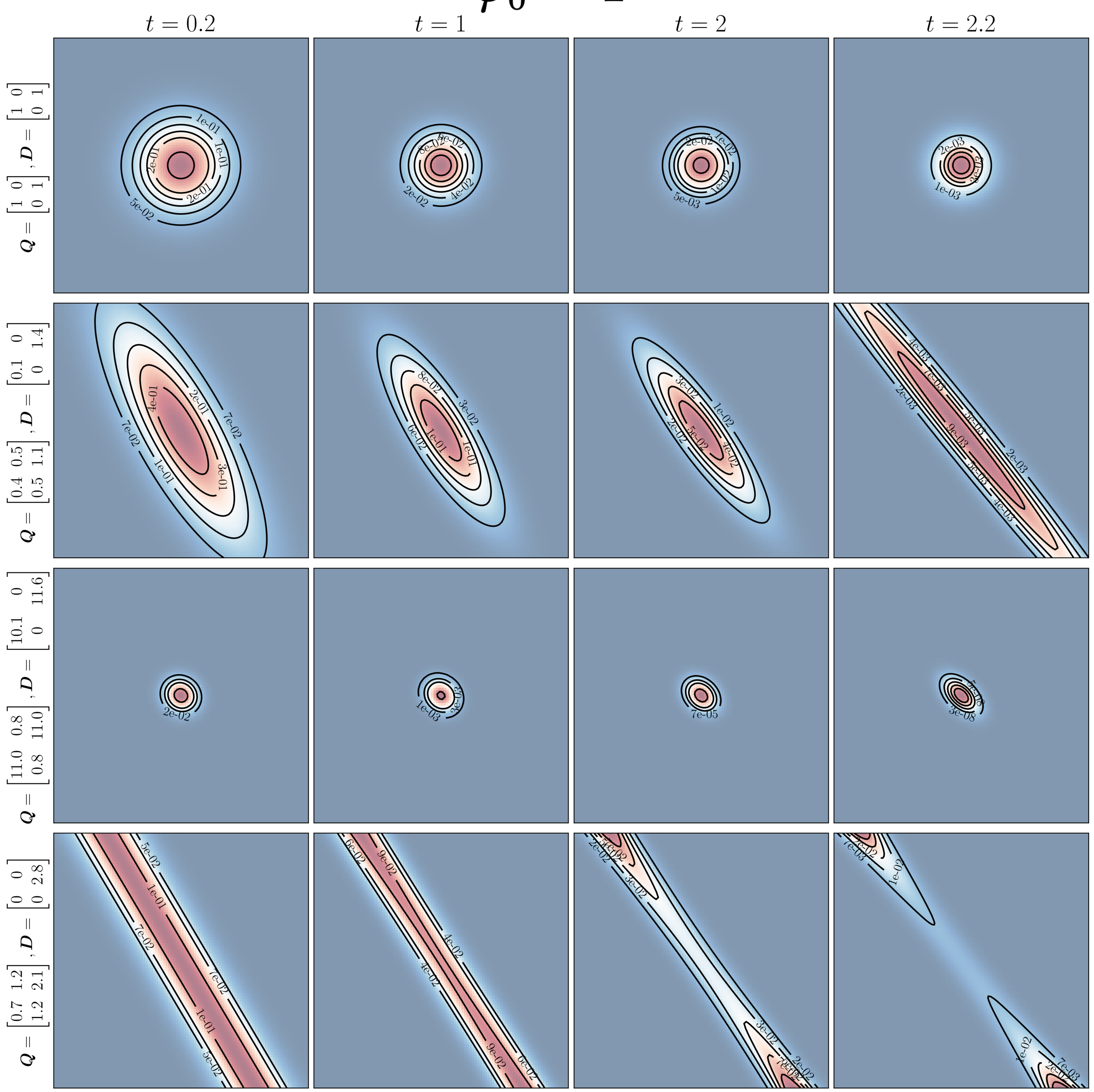
$\boldsymbol{Q} = \boldsymbol{I}$ recovers the multivariate Mehler kernel in quantum harmonic oscillator

SB with Quadratic State Cost: $q(x) = x^\top Q x, Q \geq 0$

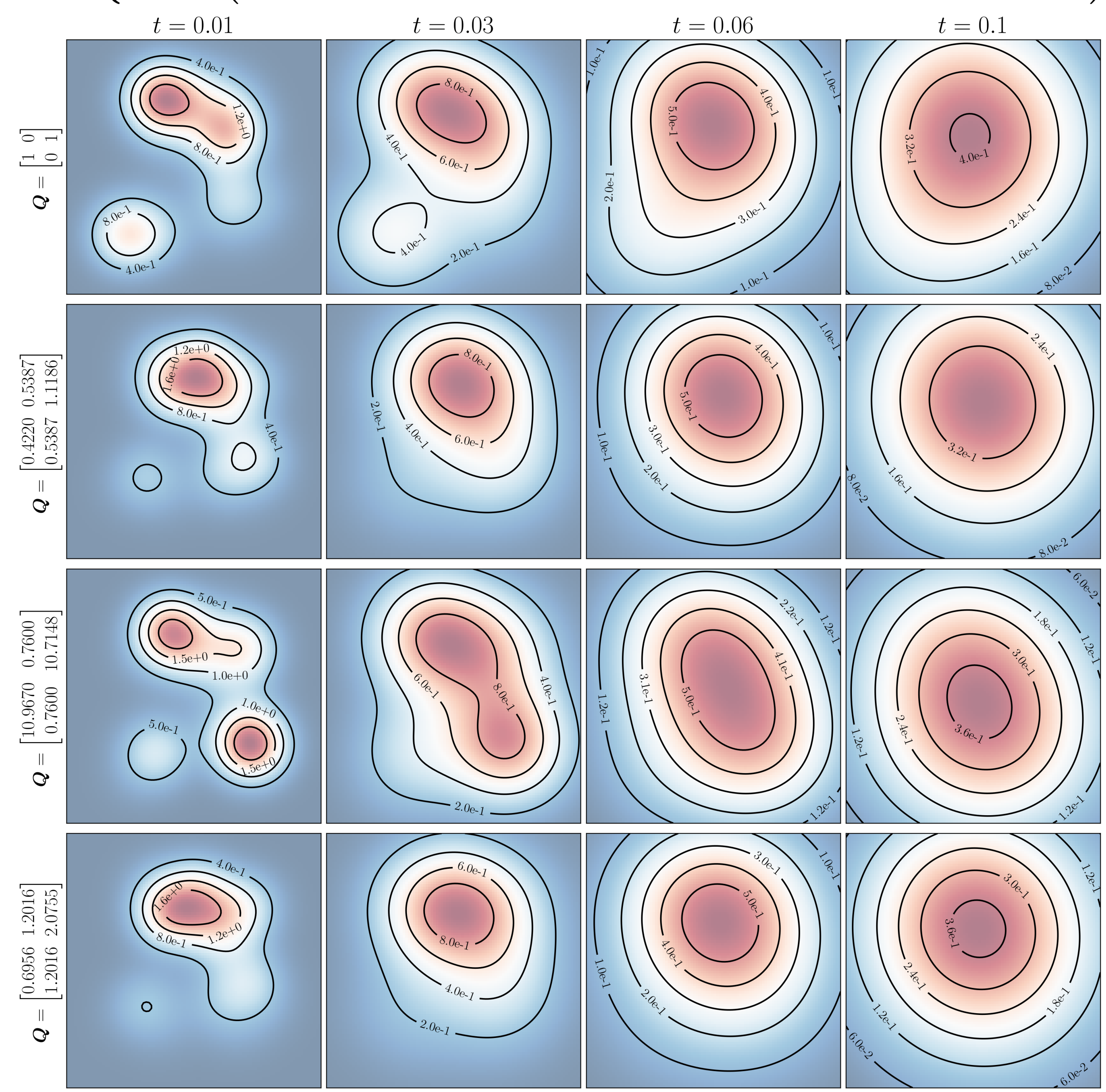
Thm. $\kappa(0, \boldsymbol{y}; t, \boldsymbol{z}) = \underbrace{\kappa_+\left(0, \boldsymbol{y}_{[i_1:i_{n-p}]}; t, \boldsymbol{z}_{[i_1:i_{n-p}]}\right)}_{\text{derived pos def kernel in } n-p \text{ variables}} \underbrace{\kappa_0\left(0, \boldsymbol{y}_{[i_{n-p+1}:i_n]}; t, \boldsymbol{z}_{[i_{n-p+1}:i_n]}\right)}_{\text{heat kernel in } p \text{ variables}}$

Action of kernel in x coordinates

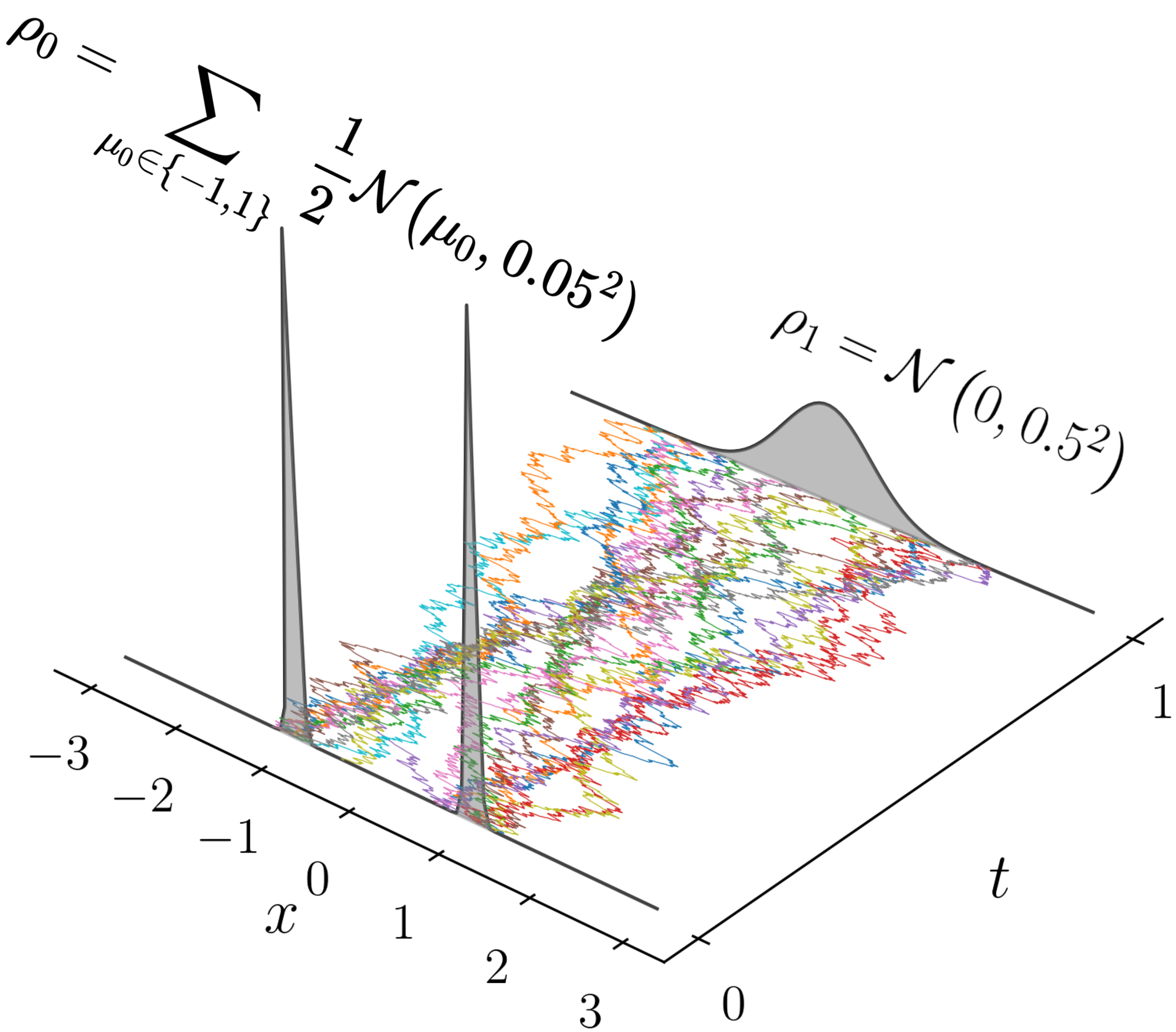
$\varphi_0 = 1$

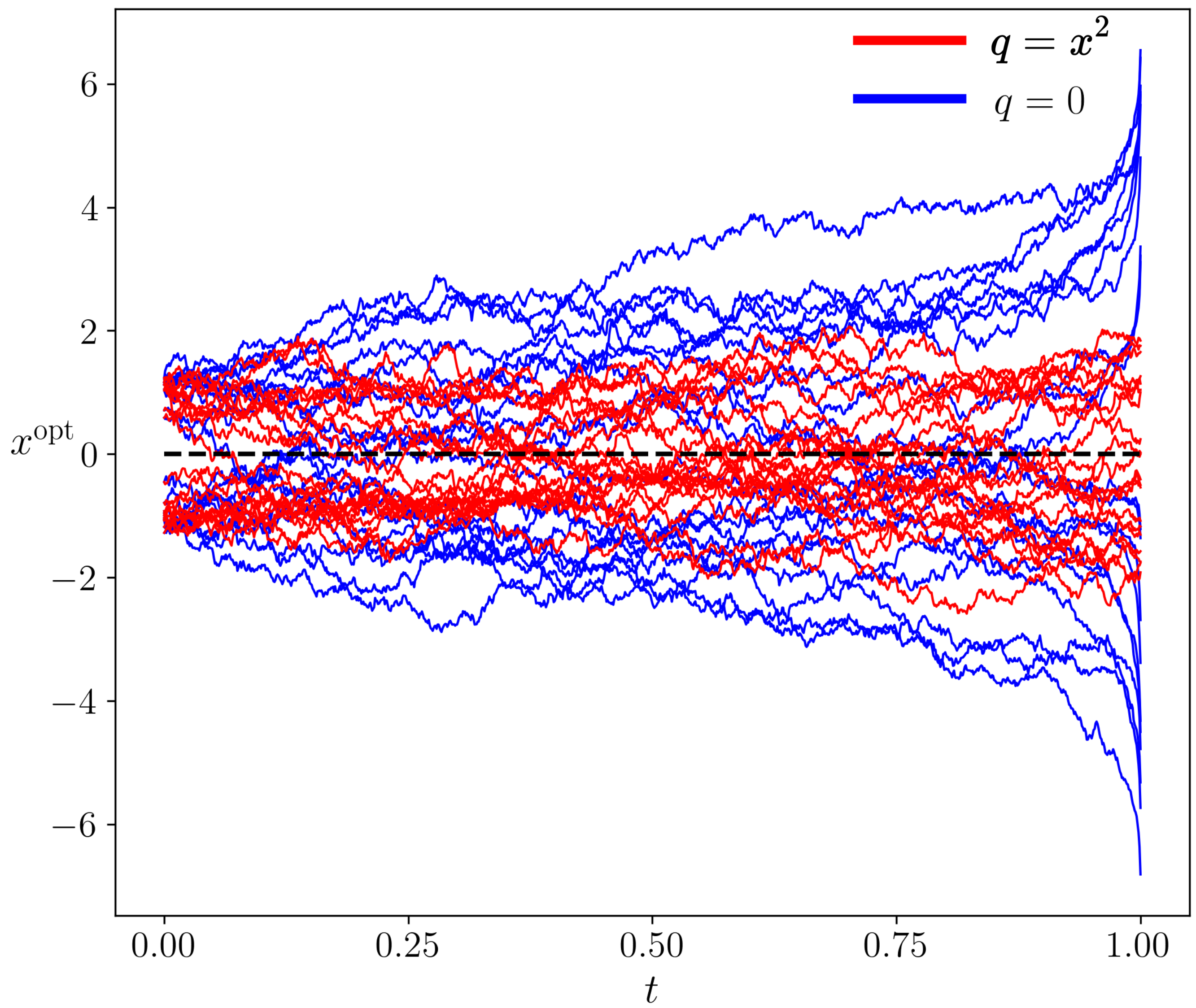


$$\varphi_0 \propto \exp\left\{-\frac{1}{25}\left(\left((10x_1-5)^2+10x_2-16\right)^2+\left(10x_1-12+(10x_2-5)^2\right)^2\right)\right\}$$

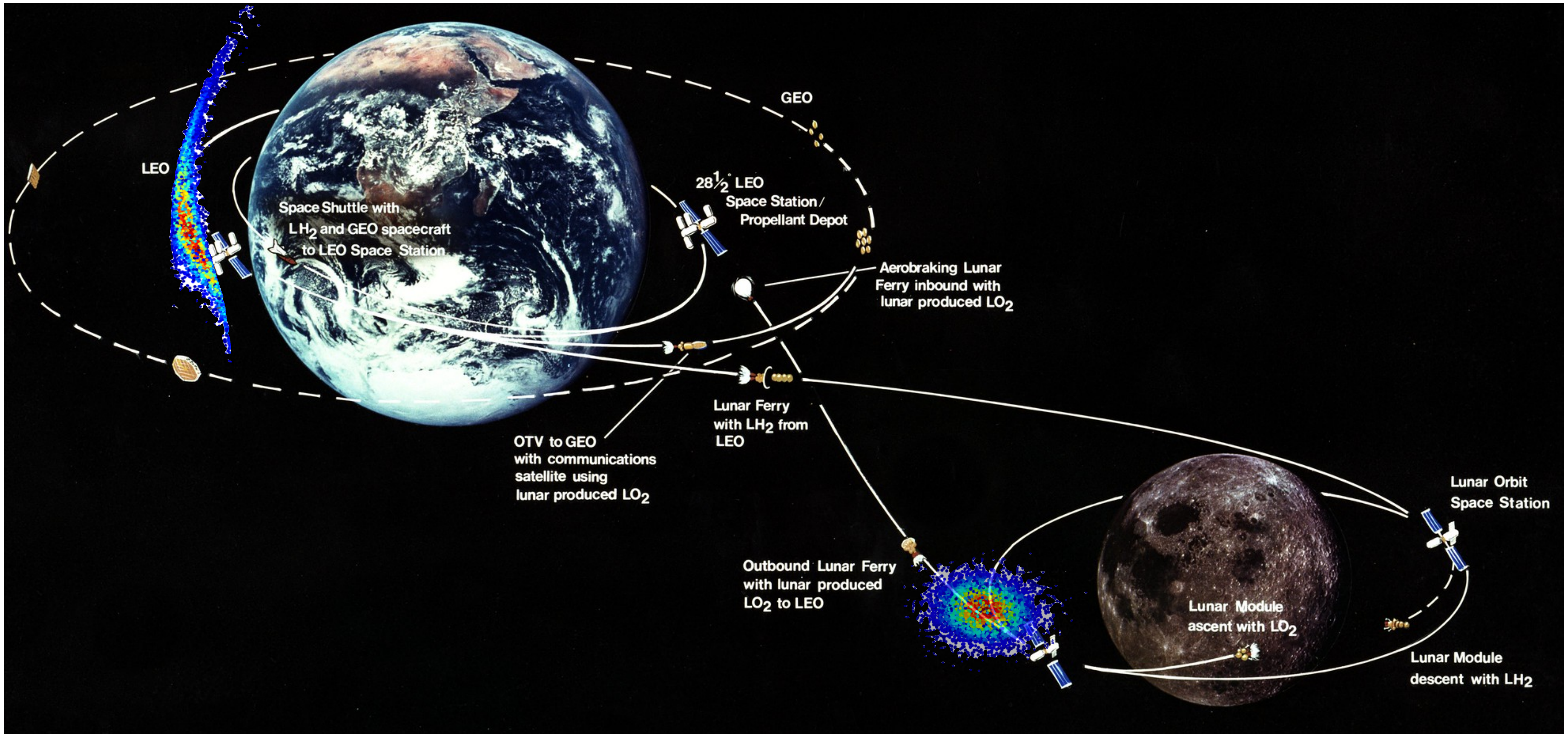


SB in 1D: with vs without Quadratic State Cost





Application: Probabilistic Lambert's Problem



3D position coordinate $\boldsymbol{r} := \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$

Find velocity control policy $\dot{\boldsymbol{r}} := \boldsymbol{v}(t, \boldsymbol{r})$ such that

$$\ddot{\boldsymbol{r}} = -\nabla_{\boldsymbol{r}} V(\boldsymbol{r}), \quad \boldsymbol{r}(t = t_0) \sim \rho_0 \text{ (given)}, \quad \boldsymbol{r}(t = t_1) \sim \rho_1 \text{ (given)}$$

Probabilistic Lambert problem is OMT

$$\underset{(\rho,\boldsymbol{v})}{\arg\inf} \quad \int_{t_0}^{t_1} \mathbb{E}_\rho\left[\frac{1}{2}\|\boldsymbol{v}\|_2^2 - V(\boldsymbol{r})\right] \mathrm{d}t$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v},$$

$$\boldsymbol{r}(t=t_0) \sim \rho_0 \text{ (given)}, \quad \boldsymbol{r}(t=t_1) \sim \rho_1 \text{ (given)}$$

$$\Updownarrow$$

$$\underset{(\rho,\boldsymbol{v})}{\arg\inf} \quad \int_{t_0}^{t_1} \mathbb{E}_\rho\left[\frac{1}{2}\|\boldsymbol{v}\|_2^2 - V(\boldsymbol{r})\right] \mathrm{d}t$$

$$\frac{\partial \rho}{\partial t} + \nabla_{\boldsymbol{r}} \cdot (\rho \boldsymbol{v}) = 0,$$

— **Liouville PDE**

$$\rho(t=t_0,\cdot) = \rho_0, \quad \rho(t=t_1,\cdot) = \rho_1$$

Connection to SB with Additive State Cost

$$\underset{(\rho,\boldsymbol{v})\in\mathcal{P}_{01}\times\mathcal{V}}{\arg\inf} \int_{t_0}^{t_1}\int_{\mathbb{R}^n}\left(\frac{1}{2}|\boldsymbol{v}|^2 - V(\boldsymbol{x})\right)\rho(\boldsymbol{x},t)\,d\boldsymbol{x}dt$$

$$\frac{\partial\rho}{\partial t} + \nabla_{\boldsymbol{r}}\cdot(\rho\boldsymbol{v}) = 0,$$ — **Liouville PDE**

$$\rho(t=t_0,\cdot)=\rho_0,\quad \rho(t=t_1,\cdot)=\rho_1$$

Lambertian SBP (L-SBP)

$$\underset{(\rho,\boldsymbol{v})\in\mathcal{P}_{01}\times\mathcal{V}}{\arg\inf} \int_{t_0}^{t_1}\int_{\mathbb{R}^n}\left(\frac{1}{2}|\boldsymbol{v}|^2 - V(\boldsymbol{x})\right)\rho(\boldsymbol{x},t)\,d\boldsymbol{x}dt$$

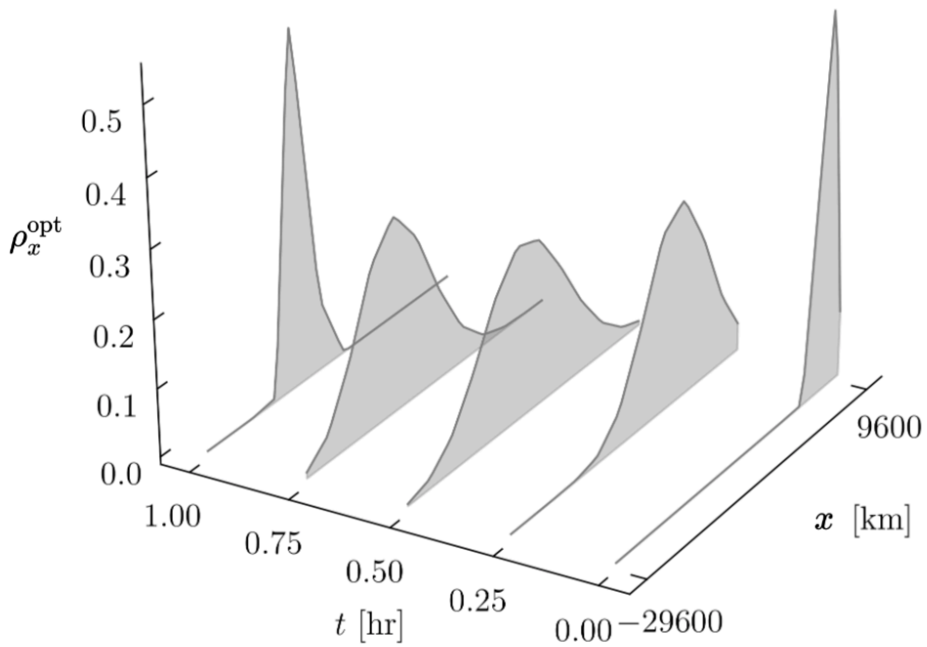
Regularization > 0

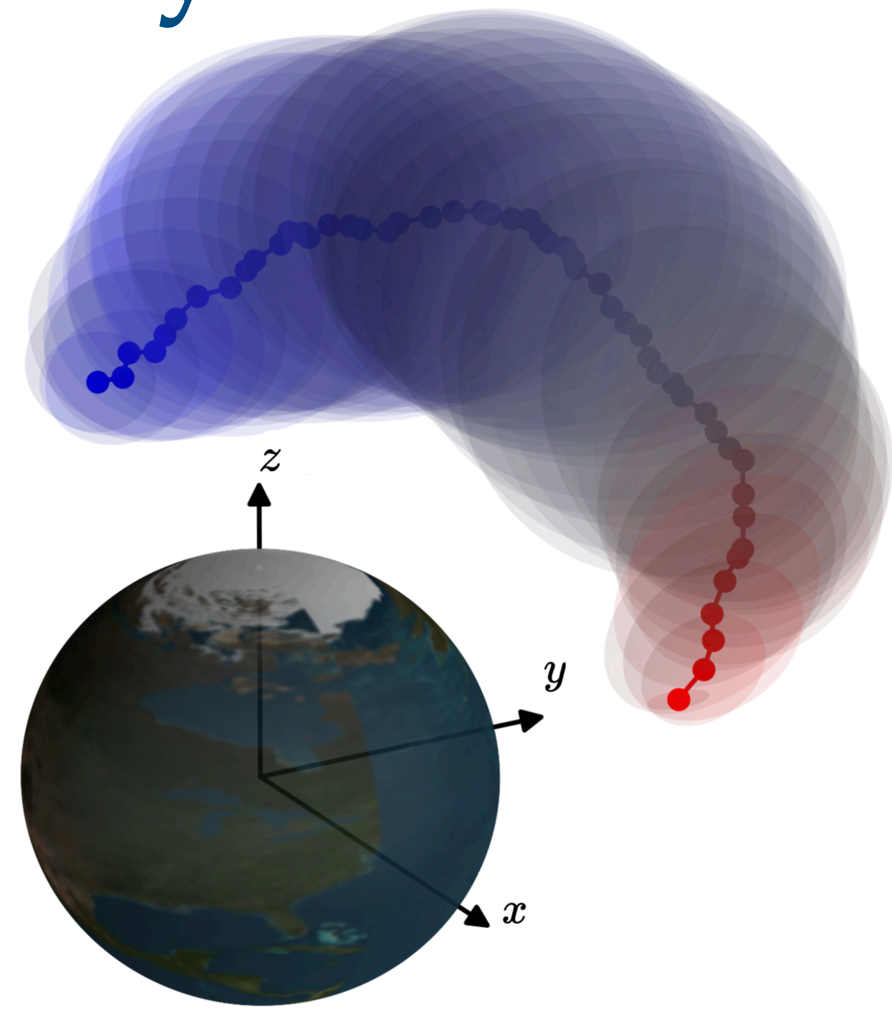
$$\frac{\partial\rho}{\partial t} + \nabla_{\boldsymbol{r}}\cdot(\rho\boldsymbol{v}) = \varepsilon\Delta_{\boldsymbol{r}}\rho,$$ — **Fokker-Planck-Kolmogorov PDE**

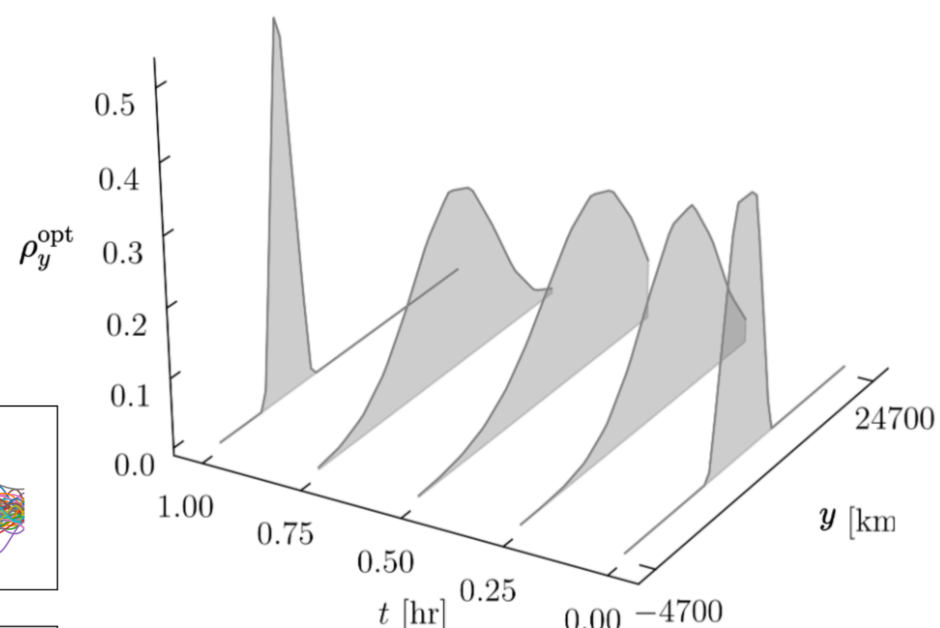
$$\rho(t=t_0,\cdot)=\rho_0,\quad \rho(t=t_1,\cdot)=\rho_1$$

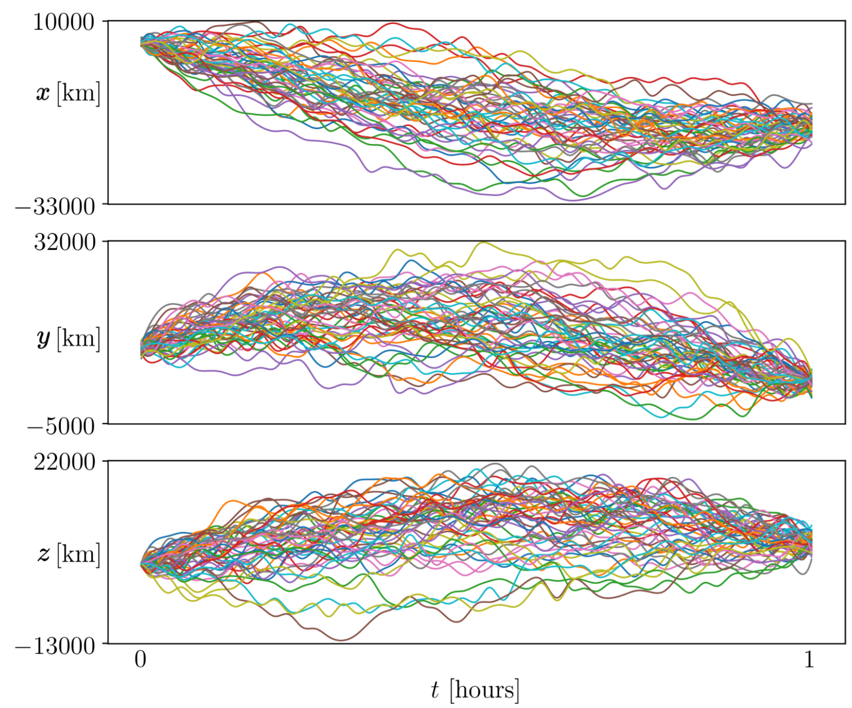
Low Earth Orbit Transfer Case Study

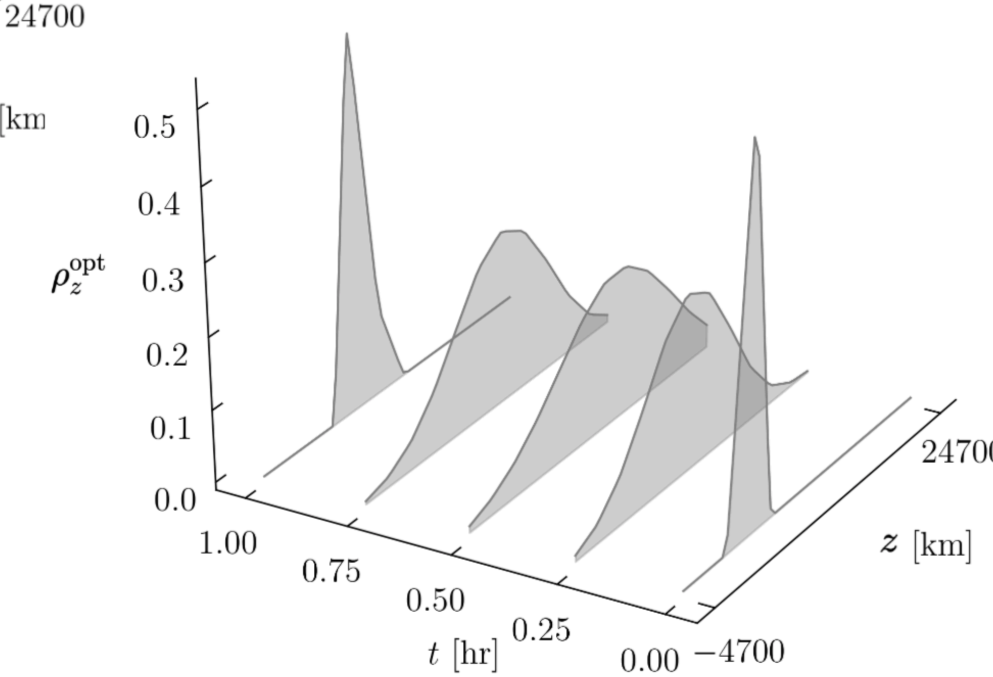
Univariate marginals for optimally controlled joint PDFs











Outlook

- Explosion of SB research in ML / AI + Control

- Lots of mathematics, algorithms, and applications to be done

- Growing interdisciplinary community

Thank You

Support:

NSF
CITRIS PEOPLE AND ROBOTS
Ford
Lawrence Livermore National Laboratory