Some details for Lec. 7, slides #11, 12 :

Factoring: $\left[1 - r^2(1-x)(1 - rx + rx^2)\right]$

$$= \left(x - \frac{r-1}{r}\right)\left(a x^2 + b x + c\right) \quad \text{----} (*)$$

to be determined

Expanding the left hand side of $(*)$:

$$r^3 x^3 - 2 r^3 x^2 + r^2(1+r)\, x + (1-r^2) \quad \text{----} (i)$$

Expanding the right hand side of $(*)$:

$$a x^3 + \left(b - a \cdot \frac{r-1}{r}\right) x^2 + \left(c - b \cdot \frac{r-1}{r}\right) x - \frac{c(r-1)}{r} = 0 \quad \text{----} (ii)$$

next pg.

Equate the coefficients of like powers of x from (i) and (ii), to obtain :

$$\boxed{a = r^3}$$

$$b - a \cdot \frac{r-1}{r} = -2r^3 \quad \longrightarrow \quad \boxed{b = -r^2(1+r)}$$

$$c - b \cdot \frac{r-1}{r} = r^2(1+r) \quad \longrightarrow \quad \boxed{c = r(1+r)}$$

$$-\frac{c(r-1)}{r} = 1 - r^2 \quad \longrightarrow \quad \text{verified.}$$

next pg.

$\therefore$ The quadratic expression in Lec. #7, p. 12 is :

$$ax^2+bx+c = r^3x^2 - r^2(1+r)x + r(1+r)$$

$\therefore$ Period 2 points are solutions of :

$$ax^2+bx+c = 0$$

$$\Leftrightarrow \underbrace{r^3x^2 - r^2(1+r)x + r(1+r) = 0}$$

$$x = \frac{r^2(1+r) \pm \sqrt{r^4(1+r)^2 - 4 \cdot r^3 \cdot r(1+r)}}{2r^3}$$

$$= \frac{r+1 \pm \sqrt{(r-3)(r+1)}}{2r}$$

This is what we wrote in Lec. 7, p. 11