Complex Crimes

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Why This Document

- Sadly, crimes against complex numbers are common among undergraduates in courses such as signals and systems, control
- Takeaway message: check before you write
- Inspired by Crimes Against Matrices by Prof. Boyd
  `https://ee263.stanford.edu/notes/matrix_crimes.pdf`

Arithmetic Crimes: Cartesian

Consider two complex numbers $z_1 = a_1 + jb_1$, $z_2 = a_2 + jb_2$ where $j := \sqrt{-1}$

Addition/subtraction: CORRECT: $z_1 \pm z_2 = (a_1 \pm a_2) + j(b_1 \pm b_2)$

Multiplication/division: WRONG: $z_1 z_2 = (a_1 a_2) + j(b_1 b_2)$, $z_1/z_2 = (a_1/a_2) + j(b_1/b_2)$

Multiplication: CORRECT: $z_1 z_2 = (a_1 + jb_1)(a_2 + jb_2) = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$

Division: CORRECT:
$z_1/z_2 = \dfrac{a_1 + jb_1}{a_2 + jb_2} = \dfrac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)} = \dfrac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + j\dfrac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$

Arithmetic Crimes: Cartesian Examples

- Let $z_1 = 2 + j3$, $z_2 = 1 - j$
- CORRECT: $z_1 + z_2 = 3 + j2$
- CORRECT: $z_1 - z_2 = 1 + j4$
- CORRECT: $z_1 z_2 = (2 + j3)(1 - j) = 2 \cdot 1 + 2 \cdot (-j) + (j3) \cdot 1 + (j3) \cdot (-j) = 5 + j$
- CORRECT:
$$z_1/z_2 = \frac{2 + j3}{1 - j} = \frac{(2 + j3)(1 + j)}{(1 - j)(1 + j)} = \frac{2 \cdot 1 + 2 \cdot j + (j3) \cdot 1 + (j3) \cdot j}{1 + 1} = -\frac{1}{2} + j\frac{5}{2}$$

Arithmetic Crimes: Polar Addition/Subtraction

Consider two complex numbers $z_1 = r_1 e^{j\theta_1}$, $z_2 = r_2 e^{j\theta_2}$ where $j := \sqrt{-1}$

Equivalent magnitude-phase notation: $|z_1| = r_1$, $|z_2| = r_2$, $\angle z_1 = \theta_1$, $\angle z_2 = \theta_2$

Addition/subtraction: WRONG: $z_1 \pm z_2 = (r_1 \pm r_2)e^{j(\theta_1 \pm \theta_2)}$

Equivalently WRONG: $|z_1 \pm z_2| = r_1 \pm r_2$, $\angle(z_1 \pm z_2) = \theta_1 \pm \theta_2$

Addition/subtraction: CORRECT:
$|z_1 \pm z_2| = \sqrt{(r_1 \cos\theta_1 \pm r_2 \cos\theta_2)^2 + (r_1 \sin\theta_1 \pm r_2 \sin\theta_2)^2}$

$\angle(z_1 \pm z_2) = \mathrm{atan2}\dfrac{r_1 \sin\theta_1 \pm r_2 \sin\theta_2}{r_1 \cos\theta_1 \pm r_2 \cos\theta_2}$, see `https://en.wikipedia.org/wiki/Atan2`

Arithmetic Crimes: Polar Multiplication/Division

Consider two complex numbers $z_1 = r_1 e^{j\theta_1}$, $z_2 = r_2 e^{j\theta_2}$ where $j := \sqrt{-1}$

Multiplication magnitude: CORRECT: $|z_1 z_2| = r_1 r_2$

Multiplication phase: WRONG: $\angle z_1 z_2 = \theta_1 \theta_2$, CORRECT: $\angle z_1 z_2 = \theta_1 + \theta_2$

Division magnitude: CORRECT: $|z_1/z_2| = r_1/r_2$

Division phase: WRONG: $\angle(z_1/z_2) = \theta_1/\theta_2$, CORRECT: $\angle(z_1/z_2) = \theta_1 - \theta_2$

Arithmetic Crimes: Polar Examples

- Let $z_1 = 1e^{j\pi/6}$, $z_2 = 2e^{j\pi/3}$
- CORRECT: $|z_1 \pm z_2| = \sqrt{\left(\cos\frac{\pi}{6} \pm 2\cos\frac{\pi}{3}\right)^2 + \left(\sin\frac{\pi}{6} \pm 2\sin\frac{\pi}{3}\right)^2} = \frac{1}{2}\sqrt{(\sqrt{3} \pm 2)^2 + (1 \pm 2\sqrt{3})^2}$
- CORRECT: $\angle(z_1 \pm z_2) = \text{atan2}\dfrac{\sin\frac{\pi}{6} \pm 2\sin\frac{\pi}{3}}{\cos\frac{\pi}{6} \pm 2\cos\frac{\pi}{3}} = \text{atan2}\dfrac{1 \pm 2\sqrt{3}}{\sqrt{3} \pm 2}$
- CORRECT: $|z_1 z_2| = 2$, $|z_1/z_2| = 1/2$
- WRONG: $\angle z_1 z_2 = \pi/18$, CORRECT: $\angle z_1 z_2 = \pi/2$
- WRONG: $\angle(z_1/z_2) = 1/2$, CORRECT: $\angle(z_1/z_2) = -\pi/6$ or $11\pi/6$

Miscellaneous Polar Crimes

$\angle(a + jb) = \text{atan2}(b/a)$, notice atan2 NOT atan, $\arctan$, $\tan^{-1}$ (last three are the same)

CORRECT: $\angle(1 + j) = \text{atan2}(1/1) = \pi/4$, $\angle(-1 - j) = \text{atan2}(-1/-1) = -3\pi/4$

WRONG: $\angle(1 + j) = \arctan(1/1) = \arctan(-1/-1) = \angle(-1 - j) = \pi/4$

Polar equality: WRONG: $r_1 e^{j\theta_1} = r_2 e^{j\theta_2}$ implies $r_1 = r_2$ and $\theta_1 = \theta_2$

Polar equality: WRONG: $r_1 e^{j\theta_1} = r_2 e^{j\theta_2}$ implies $r_1 = r_2$ and $\theta_1 = \theta_2 + 2k\pi$ for $k \in \mathbb{N}$

Polar equality: CORRECT: $r_1 e^{j\theta_1} = r_2 e^{j\theta_2}$ implies $r_1 = r_2$ and $\theta_1 = \theta_2 + 2k\pi$ for $k \in \mathbb{Z}$

Power and Root Crimes

de Moivre's Theorem: for positive integer n and complex $z = re^{j\theta} = r\left(\cos\theta + j\sin\theta\right)$

$$z^n = \left(re^{j\theta}\right)^n = r^n e^{jn\theta} = r^n\left(\cos n\theta + j\sin n\theta\right)$$

For positive integer n and complex $z = re^{j\theta} = r\left(\cos\theta + j\sin\theta\right)$, there are n complex nth roots of z, given by

$$z_k = r^{1/n} e^{\frac{\theta + 2\pi k}{n}} = r^{1/n}\left(\cos\frac{\theta + 2\pi k}{n} + j\sin\frac{\theta + 2\pi k}{n}\right), \quad k = 0, 1, \ldots, n-1$$

WRONG: For z complex, $|e^z| = e^{|z|}$

CORRECT: For z complex, $|e^z| = e^{\Re z} \le e^{|z|}$ $(\exp$ is increasing on real line and $\Re z \le |z|)$

Power and Root Crimes

Square root is not distributive if the domain is not nonnegative

WRONG: $1 = \sqrt{1} = \sqrt{(-1)^2} = \sqrt{-1} \cdot \sqrt{-1} = j \cdot j = j^2 = -1$ (incorrect third equality)

Power is not associative if the base is complex, i.e., $(z^a)^b \neq z^{ab}$ for complex z, real a, b

WRONG: $-j = (-j)^1 = (-j)^{2 \cdot \frac{1}{2}} = \left((-j)^2\right)^{\frac{1}{2}} = (-1)^{\frac{1}{2}} = j$ (incorrect third equality)

Power is not associative if any of the exponents is complex, i.e., $(x^a)^b \neq x^{ab}$ for real x, and either of a, b complex

WRONG: $1 = \sqrt{1} = \left(e^{j2\pi}\right)^{\frac{1}{2}} = e^{j\pi} = \cos\pi + j\sin\pi = -1$ (incorrect third equality)

Inequality Crimes

Unlike the set of real numbers, the set of complex numbers is not ordered, i.e., we cannot write inequalities between two complex numbers

WRONG: Since $2 > 1$ and $4 > 3$, therefore $2 + j4 > 1 + j3$

WRONG: $j2 > j$

WRONG: $j > -j$

Conjugate Crimes

Changing the signs of both real and imaginary parts: WRONG: $\overline{3 - j4} = -3 + j4$, CORRECT: $\overline{3 - j4} = 3 + j4$

WRONG: $\overline{2} = \overline{2 + j0} = -2$, CORRECT: $\overline{2 + j0} = 2$

Forgetting $j^2 = -1$ when clearing denominator with complex conjugate:
WRONG: $\dfrac{1}{1 + j2} = \dfrac{1 - j2}{(1 + j2)(1 - j2)} = \dfrac{1 - j2}{1^2 - 2^2} = -\dfrac{1}{3} + j\dfrac{2}{3}$

CORRECT: $\dfrac{1}{1 + j2} = \dfrac{1 - j2}{(1 + j2)(1 - j2)} = \dfrac{1 - j2}{1^2 + 2^2} = \dfrac{1}{5} - j\dfrac{2}{5}$

Mixing up rational and complex conjugates: WRONG: $\overline{\sqrt{2} + j} = -\sqrt{2} - j$