

AerE 3310: Flight Control Systems I: Fall 2026

Iowa State University

Midterm 1

Name:_____ Student ID:_____

For this exam you only need a pen/pencil. Calculators or other electronic devices are NOT allowed. Write your answers in the spaces provided. If you need more space, work on the other side of the page.

Problem 1

[(2 × 5) + (5 + 5) = 20 points]

A model for controlling a spacecraft's orientation $\theta(t)$ via torque input $u(t)$, is given by

$$J\ddot{\theta}(t) = u(t), \quad y(t) = \theta(t), \quad (1)$$

where the constant scalar J denotes the spacecraft's moment of inertia about its center of mass. The initial conditions are

$$\theta(0) = 0, \quad \dot{\theta}(0) = 0. \quad (2)$$

(a) Define a state vector $\mathbf{x}$ of appropriate size, and then rewrite the above control system in standard LTI state space form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad y = \mathbf{C}\mathbf{x} + \mathbf{D}u,$$

where you need to clearly write down all entries of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ for this system.

Solution for Problem 1(a)

Similar to Lec. 3 p. 4-8, Lec. 4 p. 4.

Defining the 2×1 state vector $\mathbf{x} := \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$, we get the standard LTI state space form

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{pmatrix} 0 \\ \frac{1}{J} \end{pmatrix}}_{\mathbf{B}} u, \quad y = \underbrace{(1 \quad 0)}_{\mathbf{C}} \mathbf{x} + \underbrace{0}_{\mathbf{D}} u.$$

(b) For the control system given by equations (1)-(2), use Laplace transform to find the output $y(t)$ when the input $u(t) = k$ for all $t \geq 0$. Your final answer should only be a function of t and J .

Hint: start by taking Laplace transform to both sides of the ODE in (1).

Solution for Problem 1(b)

Very similar to Lec. 8 p. 5-6.

Since $y(t) = \theta(t)$ and $u(t) = k$, taking Laplace transform to both sides of the ODE gives

$$J\mathcal{L}[\ddot{y}] = \mathcal{L}[k] \Rightarrow Js^2Y(s) = \frac{k}{s} \Rightarrow Y(s) = \frac{k}{Js^3} \Rightarrow y(t) = \frac{k}{J}\mathcal{L}^{-1}\left[\frac{1}{s^3}\right] = \frac{k}{2J}t^2,$$

where we used the formula given in the last page. In exam, we supplied specific value of k .

Problem 2

[1 point each $\times 5 = 5$ points]

For each of the following 5 multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. You DO NOT need to give any reasons.

- SISO stands for
 - (a) single input single output. [See Lec. 3 p. 1]
 - (b) several input several output.
 - (c) signed impulse signed output.
- Laplace transform of a function $f(t)$ is $\mathcal{L}[f(t)] := \int_0^\infty e^{-st}f(t)dt$. Here s is a
 - (a) complex variable. [See Lec. 6 p. 6]
 - (b) real variable.
 - (c) integer variable.
- If a SISO linear control system has impulse response function $h(t, \tau) = t^2 - \tau^2$, then
 - (a) it must be LTV system. [See Lec. 5 p. 8, PS02 Problem 1]
 - (b) it must be LTI system.
 - (c) it is neither LTI nor LTV system.
- A SISO LTI system excited with input $\sin t$ gives output t^4 . The same system excited with input $\cos t$ gives output t^5 . If we excite this system with input $\sin t + \cos t$, then the output will be
 - (a) $t^4 - t^5$.
 - (b) t .
 - (c) $t^4 + t^5$. [See Lec. 5 p. 8]
- The Laplace transform $F(s) = \frac{(s+1)(s-7)}{s^3+s}$
 - (a) has poles at 0, 1, -1.
 - (b) has zeros at 1, -7.
 - (c) has poles at 0, j , $-j$. [See Lec. 7 p. 1-4]

Some useful information

- If the Laplace transform $\mathcal{L}[f(t)] = F(s)$, then for zero initial conditions, we have
$$\mathcal{L}[\dot{f}] = sF(s), \mathcal{L}[\ddot{f}] = s^2F(s).$$
- $\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$ for $n = 0, 1, 2, \dots$. In particular, $\mathcal{L}[k] = \frac{k}{s}$ for any real k .
- The complex number $j = \sqrt{-1}$.