

AerE 3310: Flight Control Systems I: Fall 2026

Iowa State University

Midterm 2

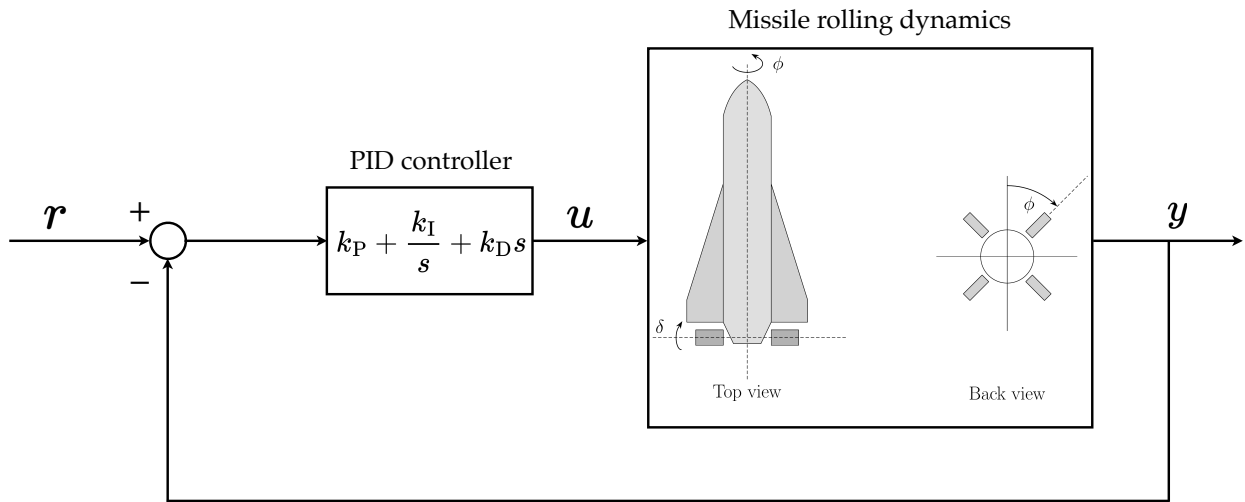
Name: _____

Student ID: _____

For this exam you only need a pen/pencil. Calculators or other electronic devices are NOT allowed. Write your answers in the spaces provided. If you need more space, work on the other side of the page.

Problem 1

[(10 + 5) + 5 = 20 points]



To control the rolling motion of a missile as shown above, the input u is the aileron deflection rate (rad/s). The output y is the roll angle ϕ (rad). The state vector $(\delta, \omega, \phi) =$ (aileron deflection angle in rad, roll angular velocity in rad/s, roll angle in rad).

The “missile rolling dynamics block” shown above relates the input u to the output y as

$$\dot{\delta} = u, \quad \dot{\omega} = -\omega + \delta, \quad \dot{\phi} = \omega, \quad y = \phi.$$

Assume zero initial conditions. Answer parts (a)-(b) below for this problem.

(a) Find the closed loop transfer function $\frac{Y}{R}$ where $Y(s)$ is the Laplace transform of the output $y(t)$, and $R(s)$ is the Laplace transform of the reference input $r(t)$. Show all calculations.

Hint: start by taking Laplace transform of the state equations to calculate $\frac{Y}{U}$, and use that to derive $\frac{Y}{R}$. You can denote the Laplace transforms: $\mathcal{L}[\delta] = \Delta$, $\mathcal{L}[\omega] = \Omega$, $\mathcal{L}[\phi] = \Phi$.

Solution for Problem 1(a)

Let $\Delta(s) := \mathcal{L}[\delta(t)]$, $\Omega(s) := \mathcal{L}[\omega(t)]$, $\Phi(s) := \mathcal{L}[\phi(t)]$, $Y(s) := \mathcal{L}[y(t)]$. Taking the Laplace transform to the state equations and using zero initial conditions, we get

$$s\Delta = U, \quad (s+1)\Omega = \Delta, \quad s\Phi = \Omega.$$

Then

$$Y = \Phi = \frac{\Omega}{s} = \frac{\Delta}{s(s+1)} = \frac{U}{s^2(s+1)} \Rightarrow \underbrace{\frac{Y}{U} = \frac{1}{s^2(s+1)}}_{\text{open loop transfer function}}.$$

Hence the closed loop transfer function

$$\begin{aligned} \frac{Y}{R} &= \frac{\left(k_P + \frac{k_I}{s} + k_D s\right) \frac{1}{s^2(s+1)}}{1 + \left(k_P + \frac{k_I}{s} + k_D s\right) \frac{1}{s^2(s+1)}} = \frac{k_P s + k_I + k_D s^2}{s^3(s+1) + (k_P s + k_I + k_D s^2)} \\ &= \frac{k_P s + k_I + k_D s^2}{s^4 + s^3 + k_D s^2 + k_P s + k_I}. \end{aligned}$$

(b) Use your answer in part (a) to determine the set of conditions that the control gain tuple (k_P, k_I, k_D) need to satisfy for closed loop stabilization.

Solution for Problem 1(b)

The denominator polynomial for $\frac{Y}{R}$ in part (a) is a degree four monic polynomial. To that, applying the necessary and sufficient conditions for the stability given in last page, we find that for closed-loop stabilization, the control gain tuple (k_P, k_I, k_D) must be in the set

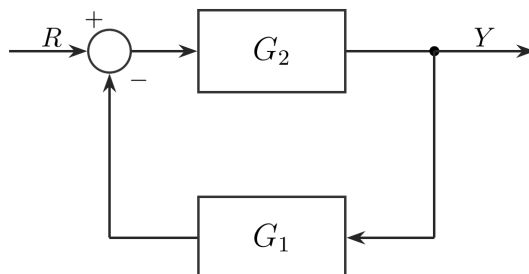
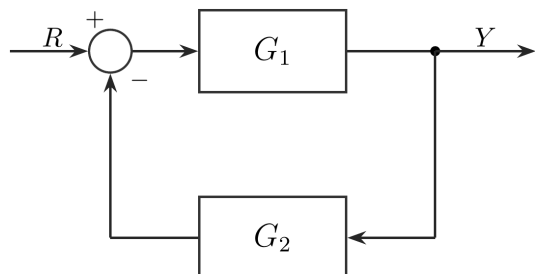
$$\left\{ (k_P, k_I, k_D) \in \mathbb{R}^3 \mid k_P > 0, k_I > 0, k_D > 0, k_P k_D - k_P^2 - k_I > 0 \right\}.$$

If PI controller: then the $\frac{Y}{R}$ transfer function expression in 1(a) specializes to the $k_D = 0$ case. In particular, then the denominator polynomial becomes a cubic with s^2 coefficient equal to zero, which makes it fail the necessary conditions for closed loop stability. So for this system, a PI controller cannot achieve closed loop stabilization.

Problem 2[1 point each $\times 5 = 5$ points]

For each of the following 5 multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. You DO NOT need to give any reasons.

- For unequal proper $G_1(s)$ and $G_2(s)$, consider the closed loop transfer functions $\frac{Y}{R}$ for the following two interconnections.



If one of these interconnections is stable, then the other

- (a) must also be stable. [similar to Lec. 15 p. 3: if $G_1 = \frac{N_1}{D_1}$, $G_2 = \frac{N_2}{D_2}$, then both $\frac{G_1}{1 + G_1 G_2} = \frac{N_1 D_2}{N_1 N_2 + D_1 D_2}$ and $\frac{G_2}{1 + G_1 G_2} = \frac{N_2 D_1}{N_1 N_2 + D_1 D_2}$ have the same denominator polynomial]
- (b) must be unstable.
- (c) may or may not be stable.

- Final Value Theorem (FVT) cannot be applied to which of the following?

- (a) $Y(s) = \frac{1}{s}$.
- (b) $Y(s) = \frac{1}{s(s-1)}$. [Lec. 09 p. 4, PS03 prob. 1(e), there is a closed RHP pole at +1.]
- (c) $Y(s) = \frac{1}{s(s+1)}$.

- Final Value Theorem (FVT) cannot be applied to which of the following?

- (a) $Y(s) = \frac{1}{s^2}$. [Lec. 09 p. 4, PS03 prob. 1(e), there is repeated pole at 0.]
- (b) $Y(s) = \frac{1}{s(s+1)}$.
- (c) $Y(s) = \frac{1}{(s+1)^2}$.

- MATLAB command `tf([1,-1],[1,2,3])` creates the transfer function

(a) $\frac{s+1}{s^2+2s+3}$.

(b) $\frac{1-s}{1+2s+3s^2}$.

(c) $\frac{s-1}{s^2+2s+3}$. [[Lecture10MATLABDemo.m](#)]

- The number of times root locus intersects the imaginary axis is

(a) always even. [[Lec. 20 p. 4](#)]

(b) always odd.

(c) either even or odd depending on the specific loop transfer function.

- If the disturbance W to output Y transfer function $G_{W \rightarrow Y}(s)$ is stable, then guaranteeing unit step disturbance rejection is same as guaranteeing

(a) $\lim_{s \rightarrow 0} G_{W \rightarrow Y}(s) = 0$. [[Lec. 17 p. 4-7](#), [Lec. 18 p. 6](#), [PS06 prob. 1\(e\)](#)]

(b) $\lim_{s \rightarrow 0} G_{W \rightarrow Y}(s) = 1$.

(c) $\lim_{s \rightarrow 0} G_{W \rightarrow Y}(s) = +\infty$.

Some useful information

- If the Laplace transform $\mathcal{L}[f(t)] = F(s)$, then for zero initial conditions, we have $\mathcal{L}\left[\dot{f}\right] = sF(s)$.

- For negative feedback, the closed loop transfer function = $\frac{\text{forward gain}}{1 + \text{loop gain}}$.

- The necessary and sufficient conditions for a degree four monic polynomial $s^4 + as^3 + bs^2 + cs + d$ to be stable is

$$a > 0, \quad b > 0, \quad c > 0, \quad d > 0, \quad abc - c^2 - a^2d > 0.$$

- Final Value Theorem (FVT) when applicable, says $\lim_{t \uparrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s)$.