

Aero 320: Numerical Methods

Lab Assignment 17

Fall 2013

Problem 1

Application of numerical differentiation: solving Partial Differential Equations (PDEs)

Consider the one dimensional heat equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad 0 < t < T,$$

that describes the evolution of temperature $u(x, t)$ along a bar of length L , as a function of position x and time t . Here α denotes the thermal diffusivity of the material. Let us assume the following boundary conditions

$$\begin{aligned} u(0, t) = u(L, t) &= 0, & 0 < t < T, \\ u(x, 0) &= \phi(x), & 0 \leq x \leq L. \end{aligned}$$

Partition the space interval $[0, L]$ as

$$x_i = i\Delta x, \quad i = 0, 1, \dots, M, \quad \Delta x = \frac{L}{M}.$$

Similarly partition the time interval $[0, T]$ as

$$t_k = k\Delta t, \quad k = 0, 1, \dots, N, \quad \Delta t = \frac{T}{N}.$$

Let us introduce the notation $u_{i,k} = u(x_i, t_k)$.

- Write the *two point forward difference* approximation for the left hand side time derivative of the heat equation.
- Write the *three point central difference* approximation for the right hand side spatial derivative of the heat equation.
- Use your answer from part (a) and (b), to approximate the heat equation as

$$u_{i,k+1} = (1 - 2\lambda) u_{i,k} + \lambda(u_{i+1,k} + u_{i-1,k}), \quad \text{where } \lambda = \frac{\alpha^2 \Delta t}{(\Delta x)^2}.$$