Aero 320: Numerical Methods

Lab Assignment 9

Fall 2013

Problem 1

Matrix and vector operations

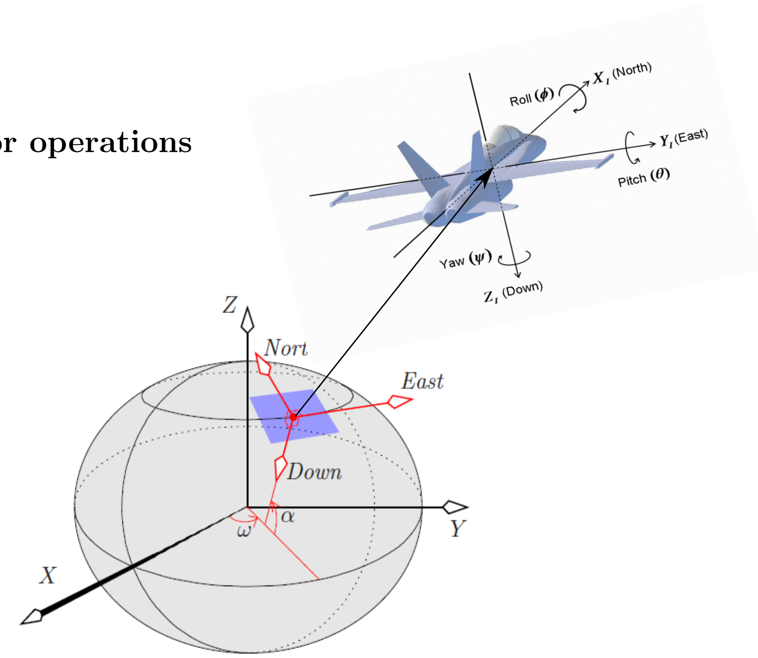

Figure 1: Rotation matrix and coordinate transformation.

In considering the movement of aircraft and space vehicles, it is frequently necessary to transform coordinate systems. The standard *inertial* North-East-Down (NED) coordinate system has the N-axis pointed north, the E-axis pointed east, and the D-axis pointed toward the center of the Earth. A second system is the vehicle's *local coordinate system*, with the i-axis straight ahead of the vehicle, the j-axis to the right, and the k-axis downward. We can transform the vector whose local coordinates are (i, j, k) to the inertial system by multiplying via *rotation matrices*:

$$\begin{pmatrix} n \\ e \\ d \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{R_\psi} \underbrace{\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}}_{R_\theta} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}}_{R_\phi} \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

(a) The rotation matrices R_ϕ, R_θ and R_ψ are of special type. What kind of matrices are they?

(b) Show that the product matrix $C = R_\psi R_\theta R_\phi$ has the same property that you find in part (a).

(c) Write a C++ program to transform the vector $(i,\ j,\ k)^\top = (2.06,\ -2.44,\ -0.47)^\top$ to the inertial system, if $\phi = 27°, \theta = 5°$, and $\psi = 72°$.

(d) Convert the vector $(n,\ e,\ d)^\top$ found in part (c) back to the local co-ordinates $(i,\ j,\ k)^\top$, using the same values for ϕ, θ, and ψ as in part (c).

(Hint: Think how to use your answer in part (b), to simplify this computation.)