Anytime Parallel Computation of Forward Reachable Tubes for Provably Safe Unmanned Aerial Systems Traffic Management

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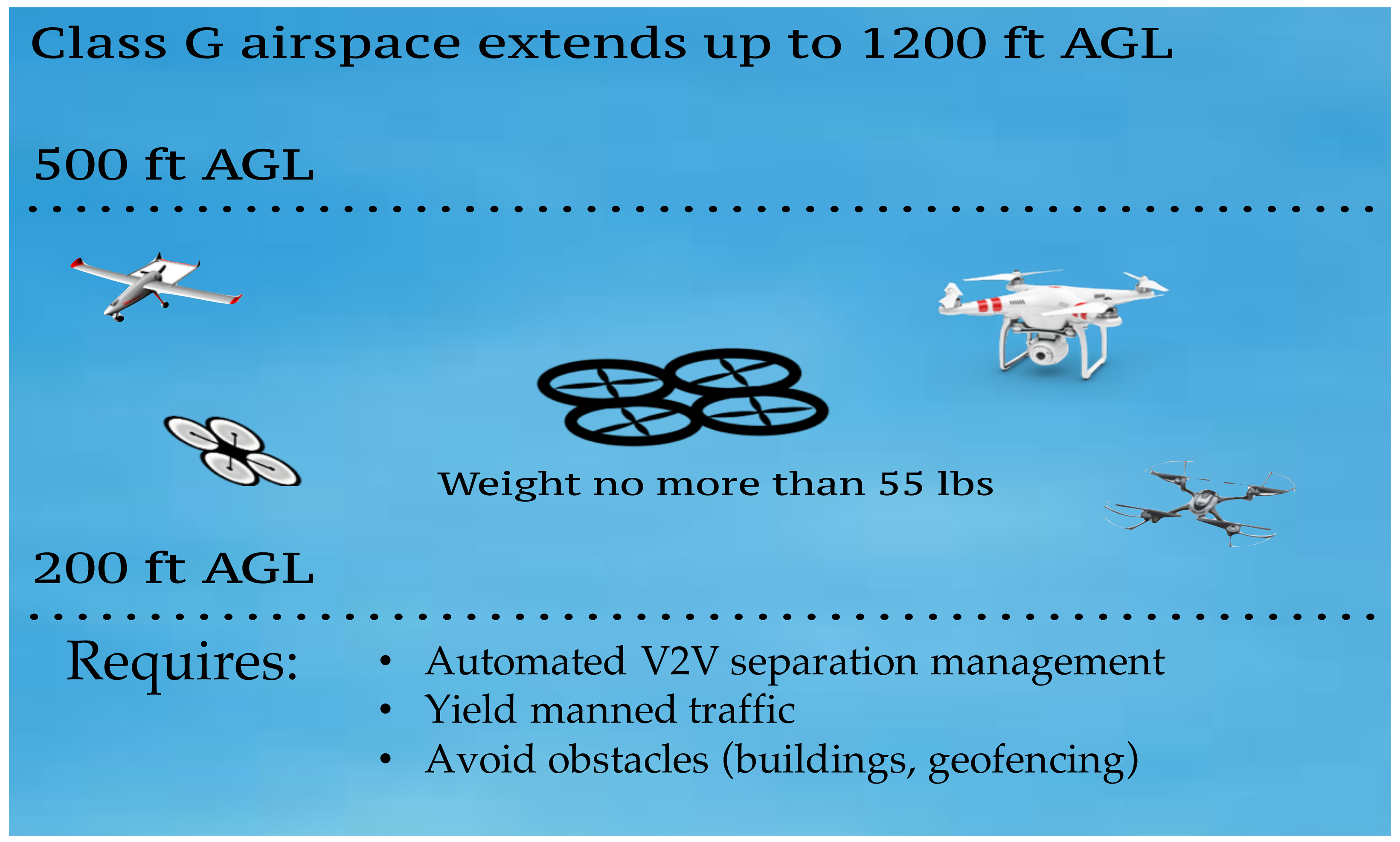
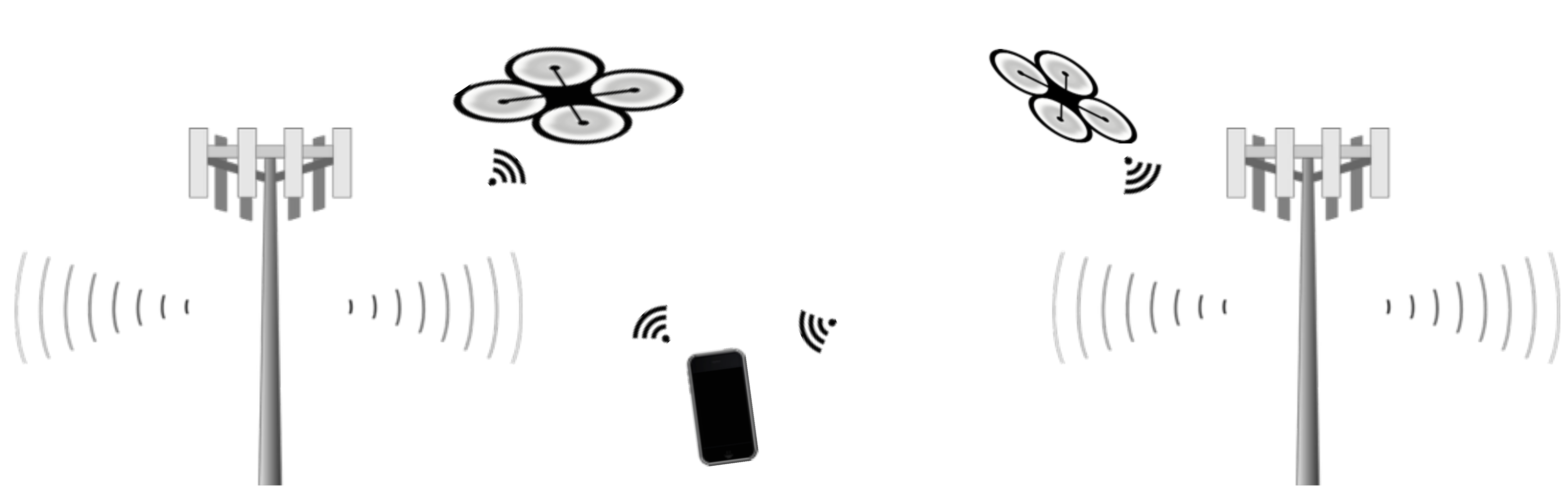
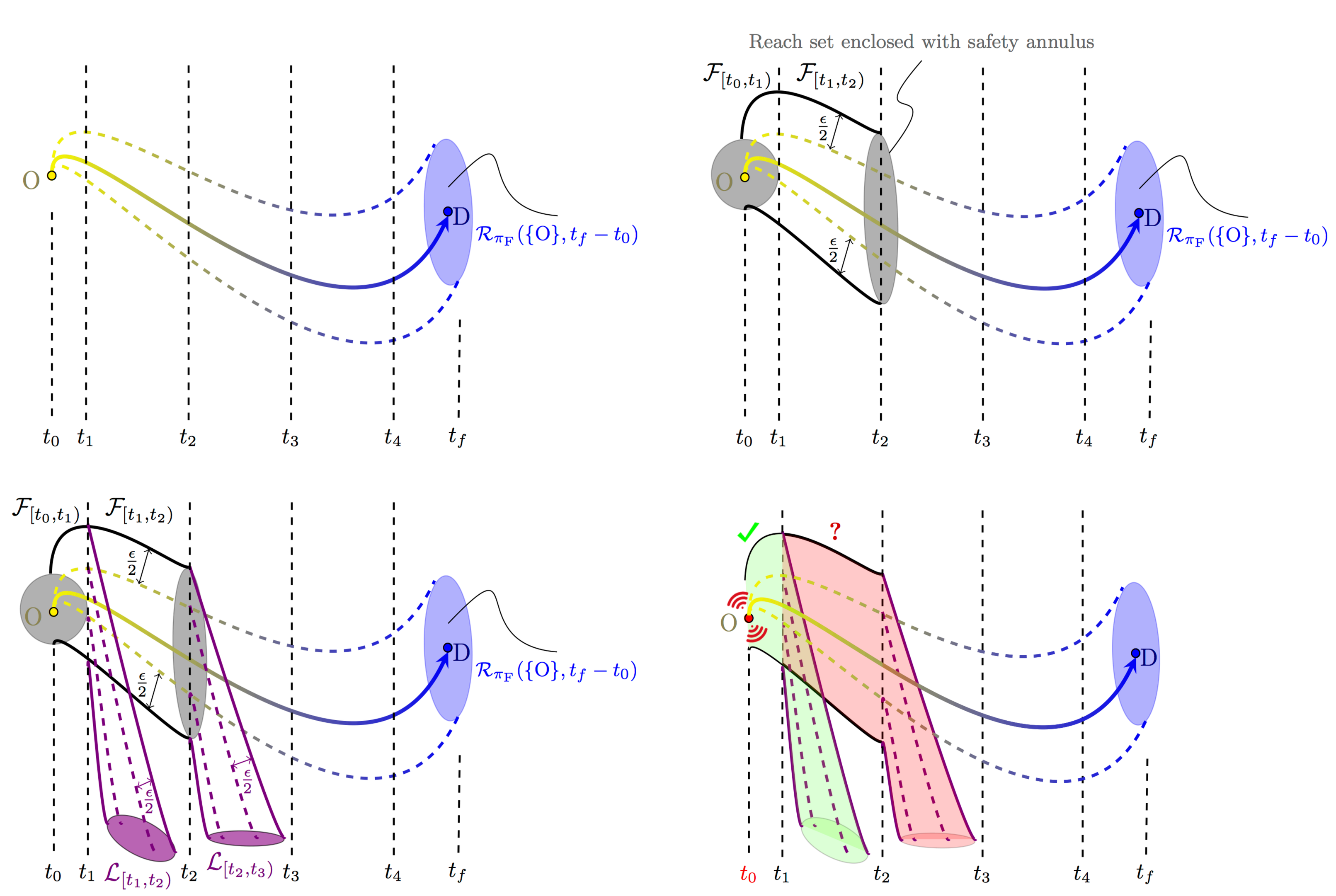
UAS Traffic Management





Motion Protocol



Snapshots of motion protocol

Algorithm for Fast Computation of Reachable Tubes

LTV closed-loop dynamics

- 12 states
- 4 controls
- 3 wind disturbances

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{\eta}(t) + \boldsymbol{G}(t)\boldsymbol{w}(t)$$

Ellipsoidal uncertainties:

$$\boldsymbol{x}(0) \in \mathcal{E}(\boldsymbol{x}_0, \mathbf{X}_0), \; \boldsymbol{\eta}(t) \in \mathcal{E}(\boldsymbol{v}(t), \boldsymbol{V}(t)),$$

$$\boldsymbol{w}(t) \in \mathcal{E}(\boldsymbol{w_c}(t), \boldsymbol{W}(t))$$

$$\mathcal{E}(\boldsymbol{x_c}, \mathbf{X}) = \{\boldsymbol{x} \in \mathbb{R}^n \mid (\boldsymbol{x}-\boldsymbol{x_c})^{\top}\mathbf{X}^{-1}(\boldsymbol{x}-\boldsymbol{x_c}) \leq 1\}$$

Compute $\hat{\mathcal{R}}$ such that $\hat{\mathcal{R}} \supseteq \mathcal{R}$, the actual reach set

$$\mathcal{E}_{\text{LJ}}\left(\text{ConvHull}(\hat{\mathcal{R}})\right) \supseteq \mathcal{E}_{\text{LJ}}(\text{ConvHull}(\mathcal{R}))$$

Kurzhanski parameterization of shape matrix:

$$\begin{aligned}\dot{\mathbf{X}}^{+}_{\ell_i(t)}(t) = &\; \boldsymbol{A}(t)\mathbf{X}^{+}_{\ell_i(t)} + \mathbf{X}^{+}_{\ell_i(t)}(t)\boldsymbol{A}(t)^{\top} \\ &+ \pi_{\ell_i(t)}(t)\mathbf{X}^{+}_{\ell_i(t)}(t) + \frac{1}{\pi_{\ell_i(t)}(t)}\boldsymbol{B}(t)\boldsymbol{V}(t)\boldsymbol{B}(t)^{\top} \\ &- \sqrt{\mathbf{X}^{+}_{\ell_i(t)}(t)}\boldsymbol{S}_{\ell_i(t)}(t)\sqrt{\boldsymbol{G}(t)\boldsymbol{W}(t)\boldsymbol{G}(t)^{\top}} \\ &- \sqrt{\boldsymbol{G}(t)\boldsymbol{W}(t)\boldsymbol{G}(t)^{\top}}\boldsymbol{S}_{\ell_i(t)}(t)^{\top}\sqrt{\mathbf{X}^{+}_{\ell_i(t)}(t)}\end{aligned}$$

Compute the Lowner-John MVOE of

$$\hat{\mathcal{R}}_{\{\ell_{i0}\}_{i=1}^{N}}(t, t_0, \mathcal{E}(\boldsymbol{x}_0, \mathbf{X}_0)) = \cap_{i=1}^{N}\mathcal{E}(\boldsymbol{x_c}(t), \mathbf{X}^{+}_{\ell_i(t)}(t))$$

Asymptotic exactness:

$$\mathcal{R}(t, t_0, \mathcal{E}(\boldsymbol{x}_0, \mathbf{X}_0)) = \bigcap_{i=1}^{\infty}\mathcal{E}(\boldsymbol{x_c}, \mathbf{X}^{+}_{\ell_i(t)}(t))$$

Inner optimization via S-procedure (SDP relaxation)

- Compute MVOE of $\cap_{i=1}^{N}\mathcal{E}_i$

Outer optimization is exact SDP

- Compute MVOE of $\cup_{k=1}^{K}\mathcal{E}_k$

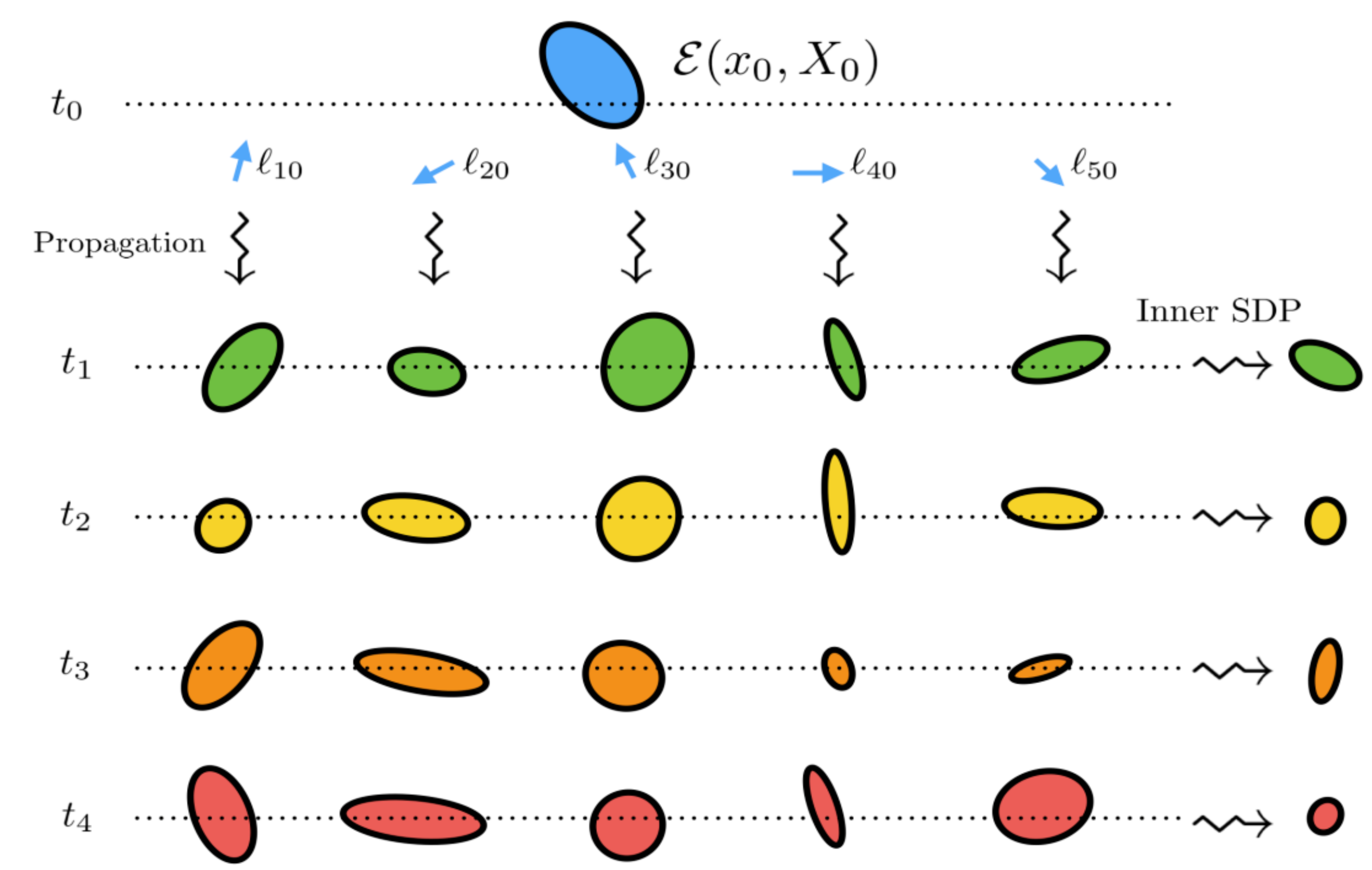
Project tube onto 3D space

$$\text{proj}\left(\mathcal{E}_{\text{LJ}}\left(\overbrace{\bigcup_{k=1}^{K}\mathcal{E}_{\text{LJ}}\underbrace{\left(\bigcap_{i=1}^{N}\mathcal{E}(\boldsymbol{x_c}(t_k), \mathbf{X}^{+}_{\ell_i(t_k)}(t_k))\right)}_{\text{inner SDP}}}^{\text{outer SDP}}\right)\right)$$

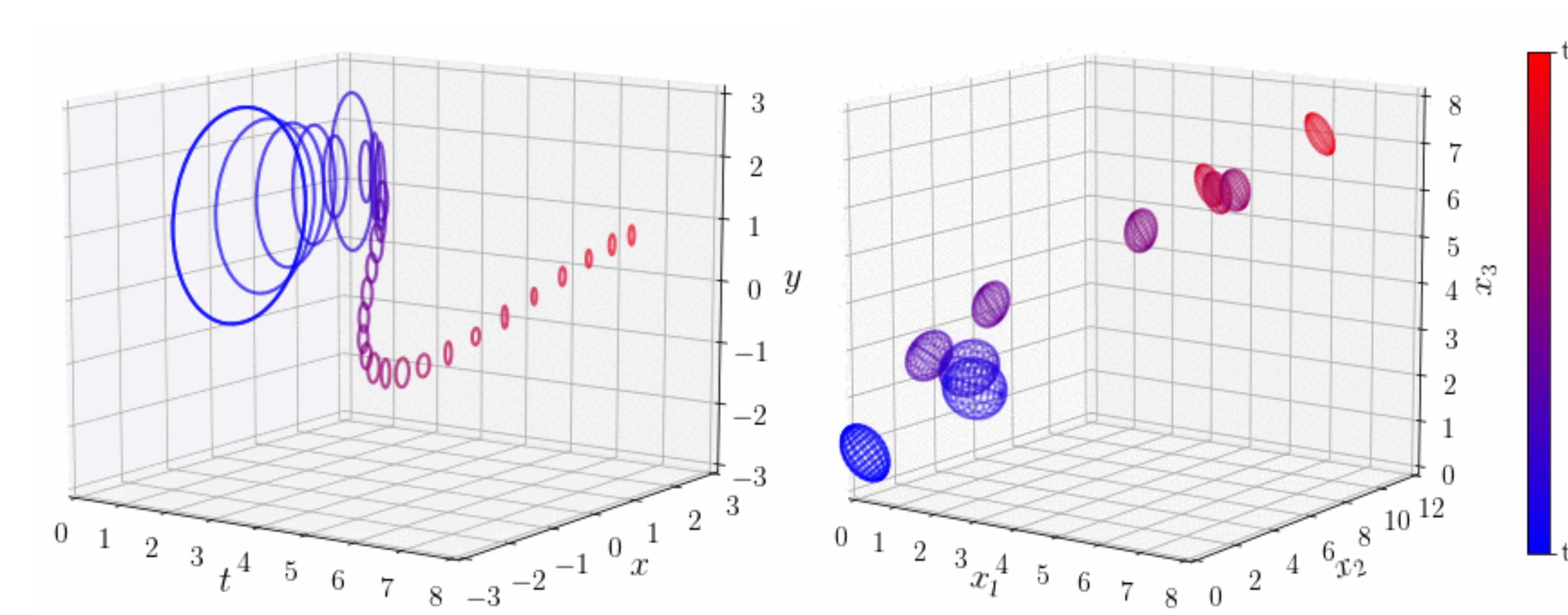
$$= \mathcal{E}_{\text{LJ}}\left(\bigcup_{k=1}^{K}\mathcal{E}_{\text{LJ}}\left(\text{proj}\left(\bigcap_{i=1}^{N}\mathcal{E}(\boldsymbol{x_c}(t_k), \mathbf{X}^{+}_{\ell_i(t_k)}(t_k))\right)\right)\right)$$

$$\subseteq \mathcal{E}_{\text{LJ}}\left(\bigcup_{k=1}^{K}\mathcal{E}_{\text{LJ}}\left(\bigcap_{i=1}^{N}\text{proj}\left(\mathcal{E}(\boldsymbol{x_c}(t_k), \mathbf{X}^{+}_{\ell_i(t_k)}(t_k))\right)\right)\right)$$

Numerical Simulations



Schematic of the anytime, parallel computational architecture



Reach set computations in two and three dimensions

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