Controllability

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Recap: linear feedback

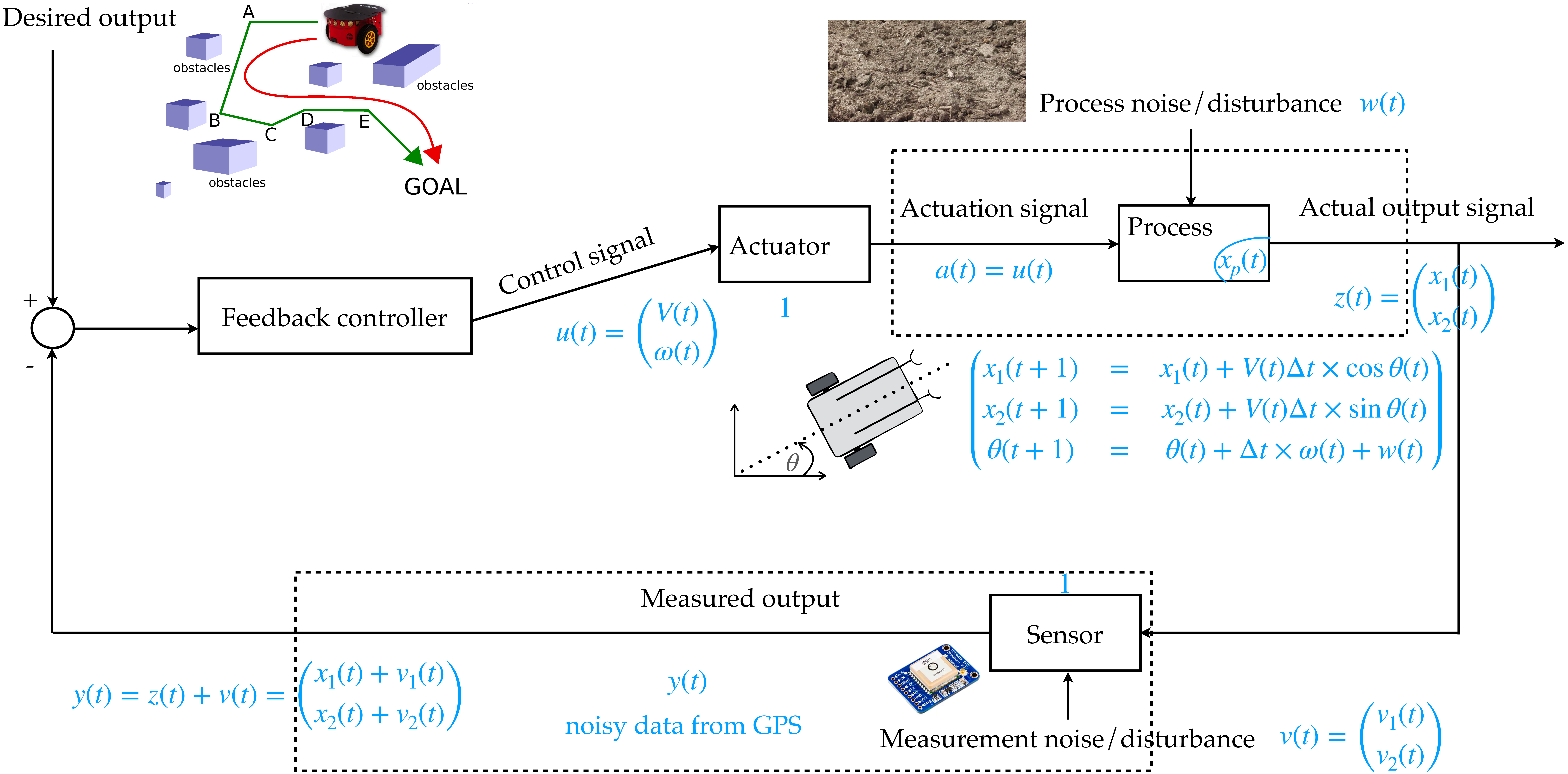
A simple control algorithm: linear feedback

Easy to implement

Generalizes proportional control

Designing the controller reduces to finding/tuning the constants/gains

Recap: wheeled mobile robot from Lecture 5



Desired output

Feedback controller

Control signal

$$u(t) = \begin{pmatrix} V(t) \\ \omega(t) \end{pmatrix}$$

Actuator

1

Actuation signal

$$a(t) = u(t)$$

Process noise / disturbance $w(t)$

Process

$x_p(t)$

Actual output signal

$$z(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

$$\begin{pmatrix} x_1(t+1) & = & x_1(t) + V(t)\Delta t \times \cos\theta(t) \\ x_2(t+1) & = & x_2(t) + V(t)\Delta t \times \sin\theta(t) \\ \theta(t+1) & = & \theta(t) + \Delta t \times \omega(t) + w(t) \end{pmatrix}$$

Measured output

Sensor

1

$$y(t) = z(t) + v(t) = \begin{pmatrix} x_1(t) + v_1(t) \\ x_2(t) + v_2(t) \end{pmatrix}$$

$y(t)$

noisy data from GPS

Measurement noise / disturbance

$$v(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}$$

MATLAB exercise: design a linear feedback controller for tracking

Ignore noise: assume $w(t) = 0$, $v_1(t) = 0$, $v_2(t) = 0$

Choose some meaningful initial condition

Choose some meaningful desired path

Use for loop over the discrete time index

Experiment with different choices of control gains

Let us look at the MATLAB code

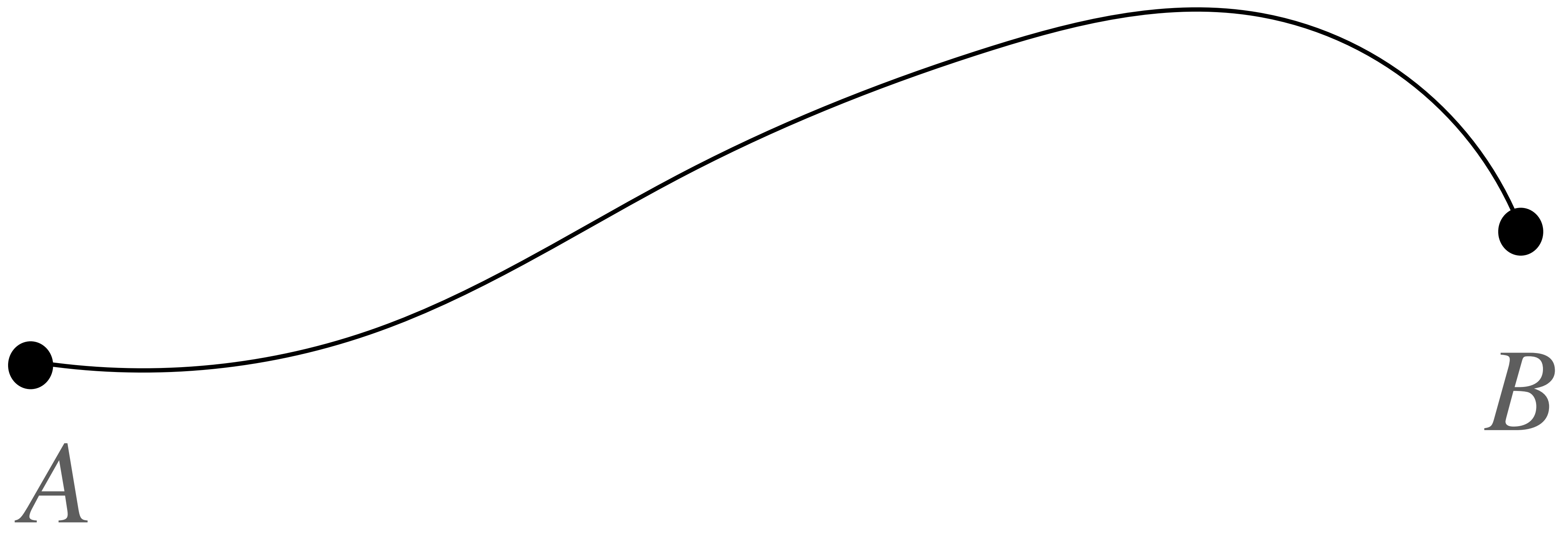
This raises an interesting question: controllability

Given two (collection of) process states, say, $A = (x_{10}, x_{20}, \theta_0)$ and $B = (x_{1T}, x_{2T}, \theta_T)$.

If there exists a time $T > 0$ and an admissible control $u(t)$ such that

$$(x_1(t=0), x_2(t=0), \theta(t=0)) = A \text{ and } (x_1(t=T), x_2(t=T), \theta(t=T)) = B$$

then the system is called **controllable**



For many systems, we can mathematically prove controllability!!

Examples:

— Two wheeled mobile robot

— Car

— Truck with multiple trailers

— Grocery shopping cart

Parking theorem: One can get out of any parking lot bigger than the size of the car.

Linear control systems in discrete time

Example: two process states (x_1, x_2) and one control u

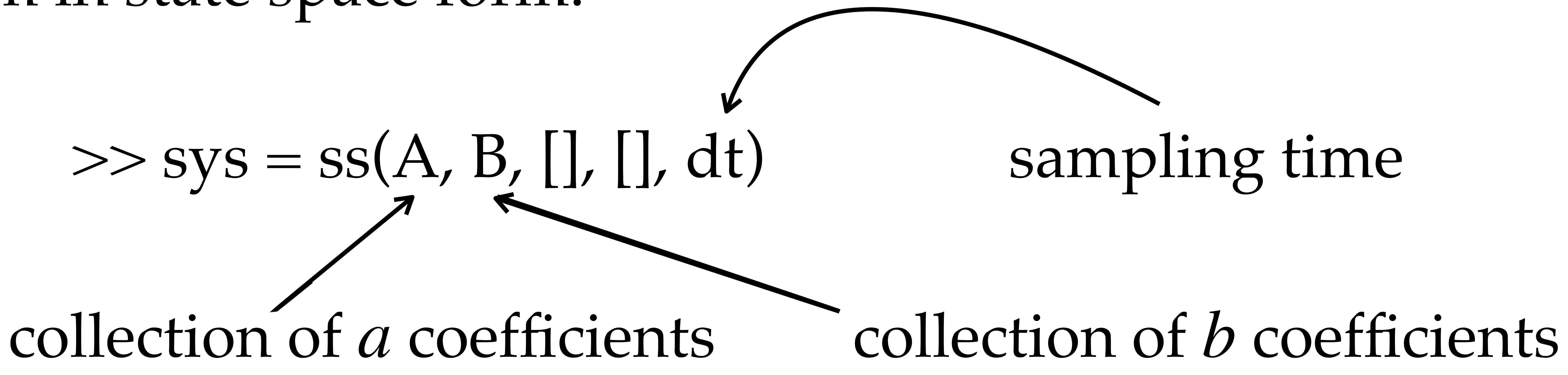
$$x_1(t+1) = a_{11}x_1(t) + a_{12}x_2(t) + b_{11}u(t)$$
$$x_2(t+1) = a_{21}x_1(t) + a_{22}x_2(t) + b_{21}u(t)$$

where the coefficients a's and b's are known constants

Can check controllability for this type of systems in MATLAB

Linear control systems in discrete time: check controllability in MATLAB

Create linear control system in state space form:



```
>> sys = ss(A, B, [], [], dt)
```

sampling time

collection of *a* coefficients

collection of *b* coefficients

Then check if the output of the following **is equal to number of process state variables**:

```
>> rank(ctrb(sys))
```

If YES, then controllable

If NO, then NOT controllable

MATLAB demo