Modeling Control Systems

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Recap: controllability

Linear feedback control algorithm given the process model in state space form

Controllability: idea and examples

Testing controllability in MATLAB for linear control systems in state space form

Discrete time control systems in state space form

Example (linear control system): two process states (x_1, x_2) and one control u

$$x_1(t+1) = a_{11}x_1(t) + a_{12}x_2(t) + b_{11}u(t)$$
$$x_2(t+1) = a_{21}x_1(t) + a_{22}x_2(t) + b_{21}u(t)$$

where the coefficients a's and b's are known constants

Example (nonlinear control system): three process states (x_1, x_2, θ) and two controls (V, ω)

$$x_1(t+1) = x_1(t) + V(t)\Delta t \times \cos\theta(t)$$
$$x_2(t+1) = x_2(t) + V(t)\Delta t \times \sin\theta(t)$$
$$\theta(t+1) = \theta(t) + \Delta t \times \omega(t) + w(t)$$

But how to write control systems in state space form

Sometimes we have: memory up to few previous time steps, one control u

Example:

$$x(t+1) + a_1 x(t) + a_2 x(t-1) = bu(t)$$

1 time step delayed state

Question: How to write the above in state space form?

But how to write control systems in state space form

Sometimes we have difference equation: memory up to few previous time steps, one control u

Example:

$$x(t+1) + a_1 x(t) + a_2 x(t-1) = bu(t)$$

Question: How to write the above in state space form?

Hint: introduce new variables:

$$x_1(t) := x(t)$$
$$x_2(t) := x(t-1) = x_1(t-1)$$

But how to write control systems in state space form

Sometimes we have difference equation: memory up to few previous time steps, one control u

Example: $$x(t+1) + a_1 x(t) + a_2 x(t-1) = bu(t)$$

Question: How to write the above in state space form?

Hint: introduce new variables:

$$x_1(t) := x(t)$$
$$x_2(t) := x(t-1) = x_1(t-1)$$

Answer:

$$x_1(t+1) = -a_1 x_1(t) - a_2 x_2(t) + bu(t)$$
$$x_2(t+1) = x_1(t)$$

Is this a linear or nonlinear control system?

Exercise: write the following in state space form

$$x(t+1) + 2x(t) - 5x(t-1) + 7x(t-2) = 3u(t)$$

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Solution: introduce new variables:

$$x_1(t) := x(t)$$
$$x_2(t) := x(t-1) = x_1(t-1)$$
$$x_3(t) := x(t-2) = x_2(t-1)$$

Therefore

$$x_1(t+1) = -2x_1(t) + 5x_2(t) - 7x_3(t) + 3u(t)$$
$$x_2(t+1) = x_1(t)$$
$$x_3(t+1) = x_2(t)$$

Is this a linear or nonlinear control system?

MATLAB exercise: controllable or not?

Recap: check controllability in MATLAB

Create linear control system in state space form:

```
>> sys = ss(A, B, [], [], dt)
```

sampling time

collection of *a* coefficients

collection of *b* coefficients

Then check if the output of the following **is equal to number of process state variables**:

```
>> rank(ctrb(sys))
```

If YES, then controllable

If NO, then NOT controllable

Exercise: write the following in state space form

$$x(t+1) + 2x^3(t) - 5x^4(t-1) + 7\sin(x(t-2)) = 3u(t) - 9u(t-1)$$

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$$x(t+1) + 2x^3(t) - 5x^4(t-1) + 7\sin(x(t-2)) = 3u(t) - 9u(t-1)$$

Solution: introduce new variables:

$$x_1(t) := x(t)$$
$$x_2(t) := x(t-1) = x_1(t-1)$$
$$x_3(t) := x(t-2) = x_2(t-1)$$

$$u_1(t) := u(t)$$
$$u_2(t) := u(t-1) = u_1(t-1)$$

Therefore

$$x_1(t+1) = -2x_1^3(t) + 5x_2^4(t) - 7\sin(x_3(t)) + 3u_1(t) - 9u_2(t)$$
$$x_2(t+1) = x_1(t)$$
$$x_3(t+1) = x_2(t)$$

Is this a linear or nonlinear control system?

Exercise: write the following in state space form

$$x(t+1) + 2x(t) - 5x(t-1) + 7x(t-2) = 3u(t)$$

$$y(t) = 4x(t) + 5x(t-1)$$

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$$x(t+1) + 2x(t) - 5x(t-1) + 7x(t-2) = 3u(t)$$

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Solution: introduce new variables:

$$x_1(t) := x(t)$$
$$x_2(t) := x(t-1) = x_1(t-1)$$
$$x_3(t) := x(t-2) = x_2(t-1)$$

Therefore

$$x_1(t+1) = -2x_1(t) + 5x_2(t) - 7x_3(t) + 3u(t)$$
$$x_2(t+1) = x_1(t)$$
$$x_3(t+1) = x_2(t)$$

sensor / measurement model

$$y(t) = 4x_1(t) + 5x_2(t)$$

MATLAB exercise: controllable or not?