Dynamics in Control Loops

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Recap: Signals in control systems

Recap: introduced signals in last lecture

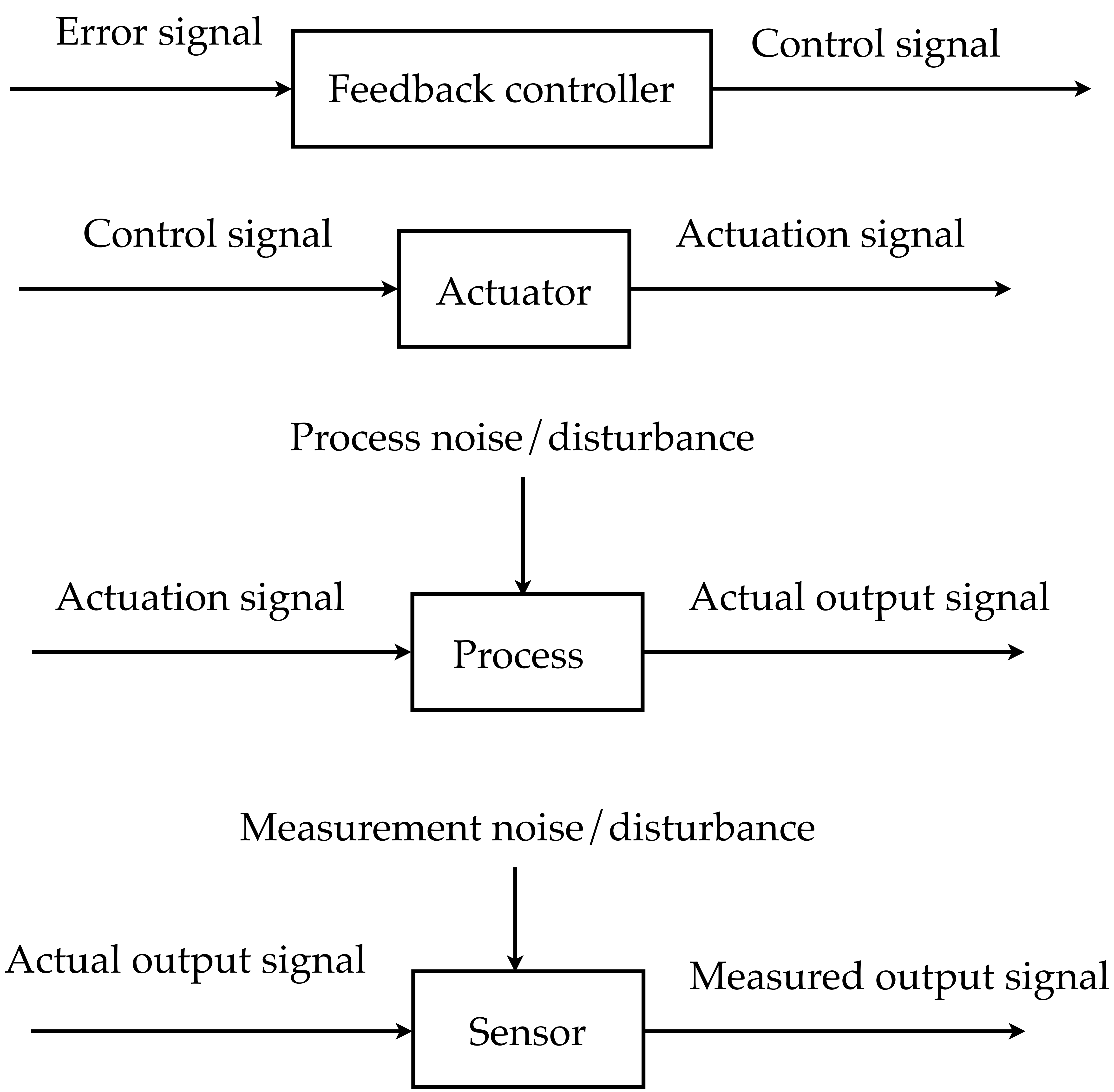
Signals: functions of time

Different types: continuous time signals, discrete time/sampled signals

Collection/vector of signals

Signals arise in control systems because different components in block diagram have **dynamics**

Different types of signals



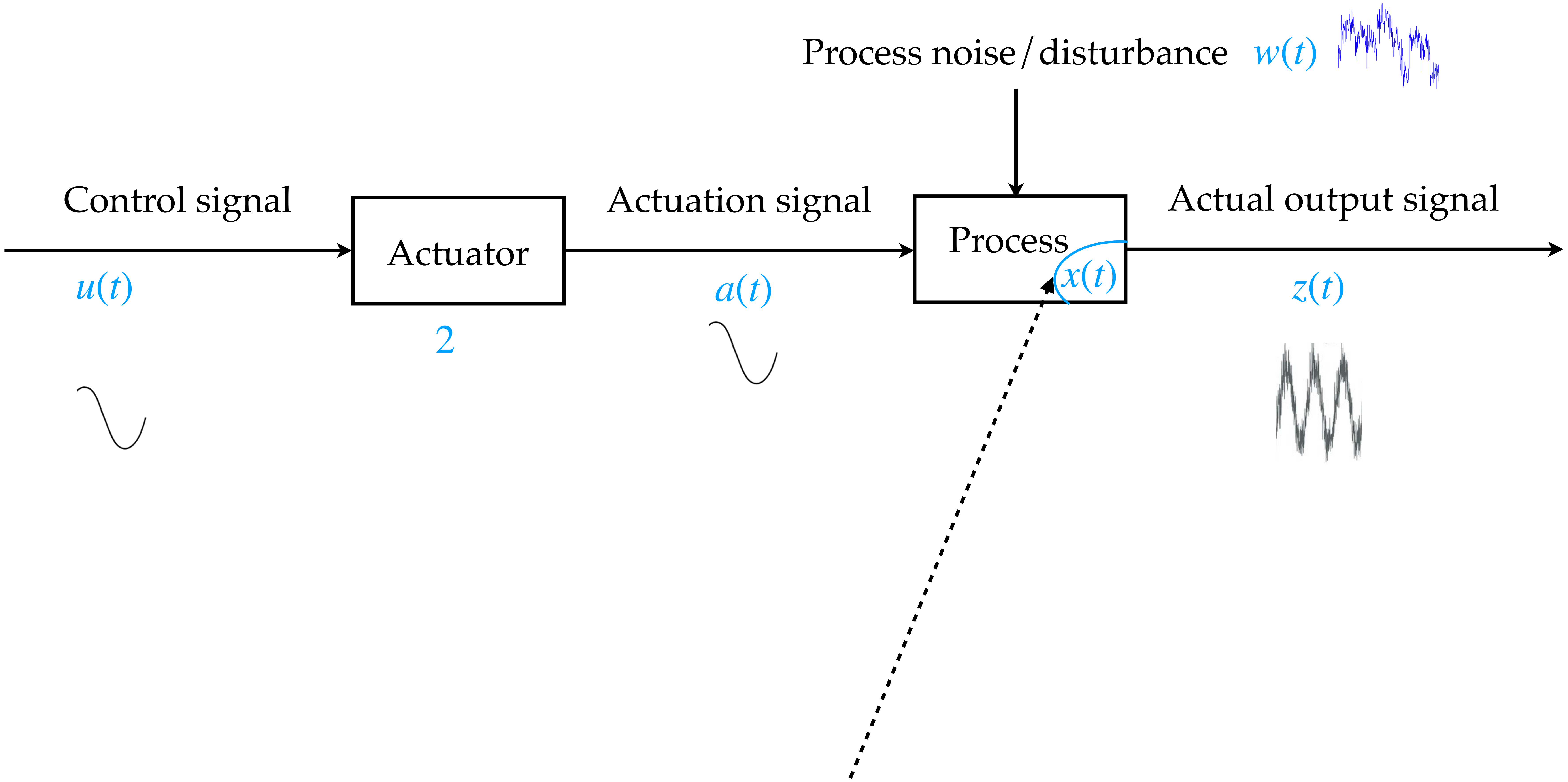
Each box has its own **dynamics**

This governs how input signals to a block gets mapped to the output signals of that box

There could also be signals **internal** to a box

Such signals are called "states" of that box

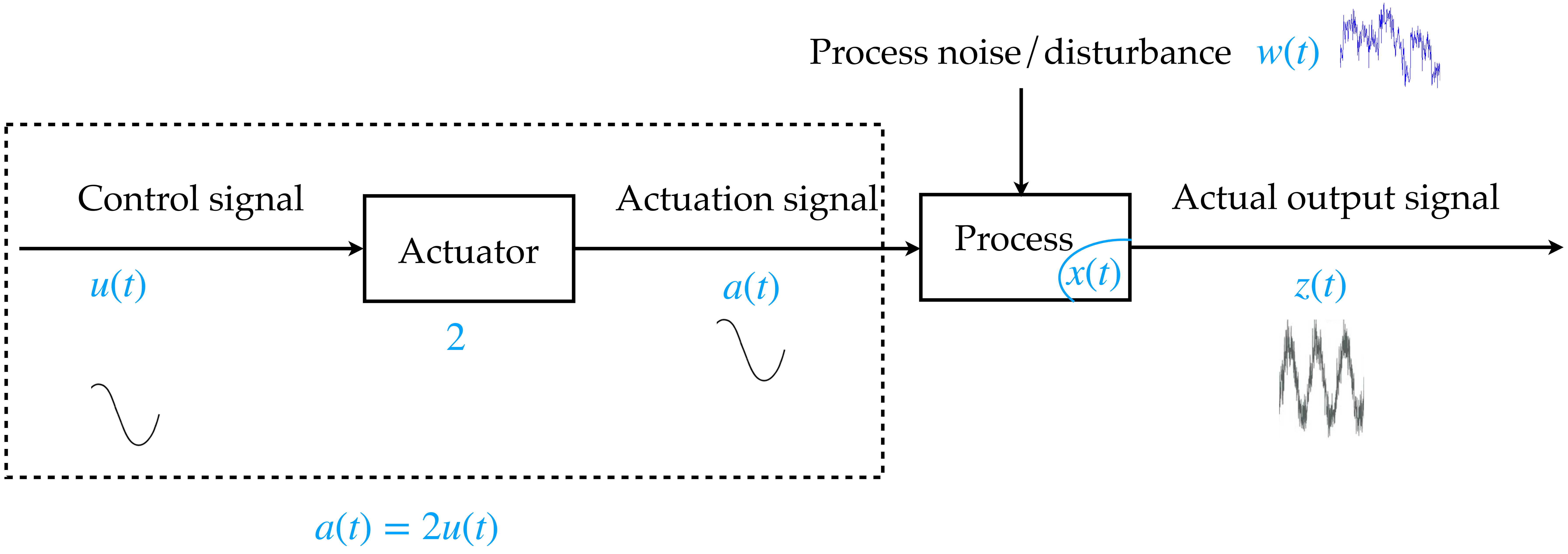
A discrete time example: process dynamics



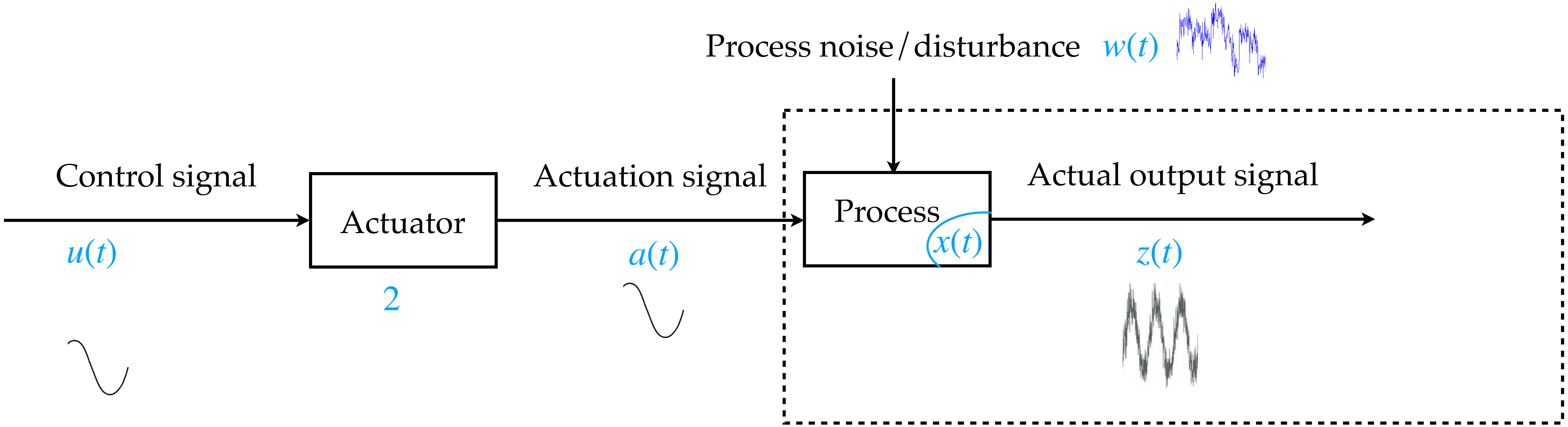
$x(t)$ is an internal signal of the process

We say $x(t)$ is the "state" of the process at time t

A discrete time example: process dynamics



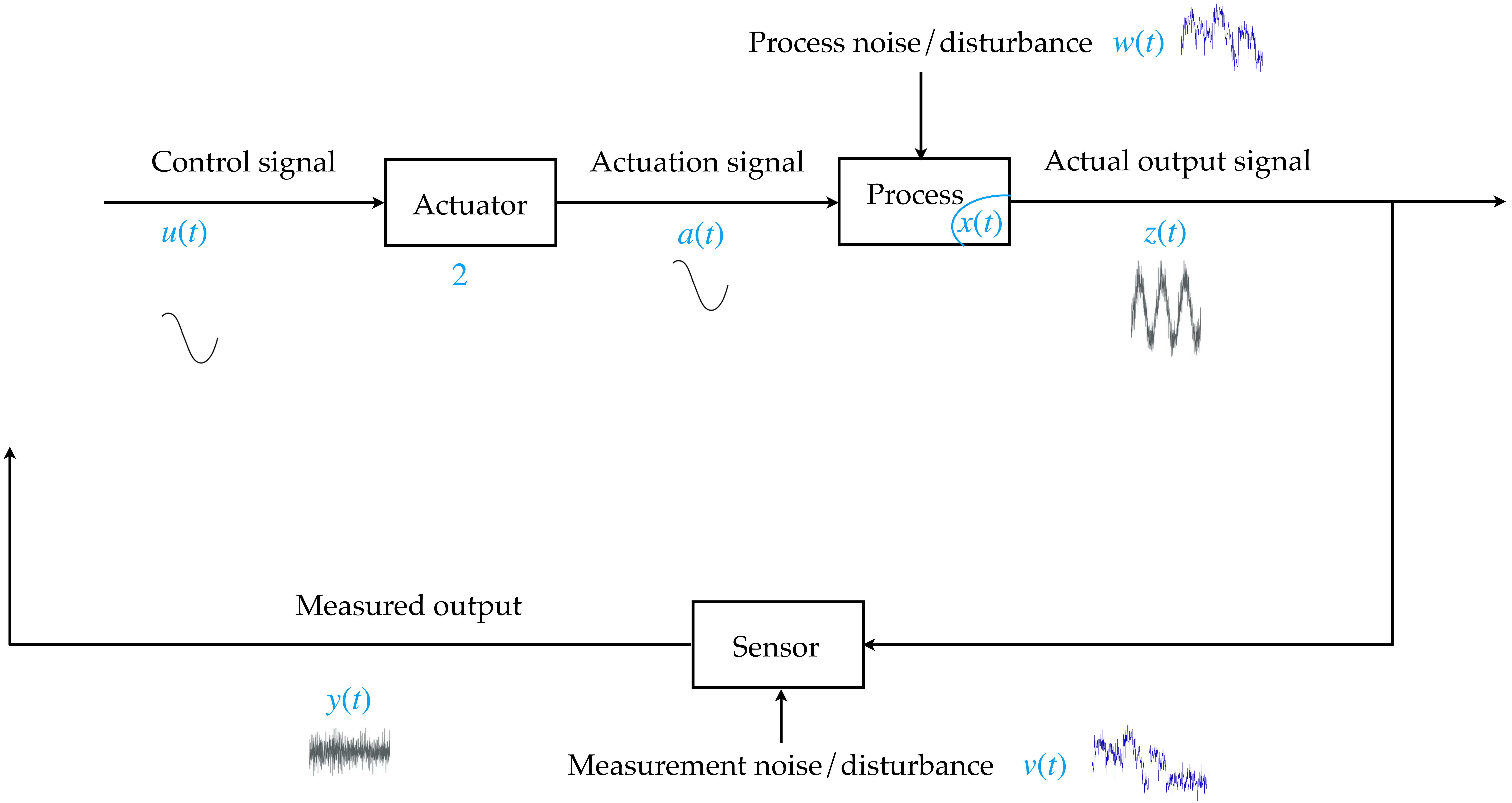
A discrete time example: process dynamics



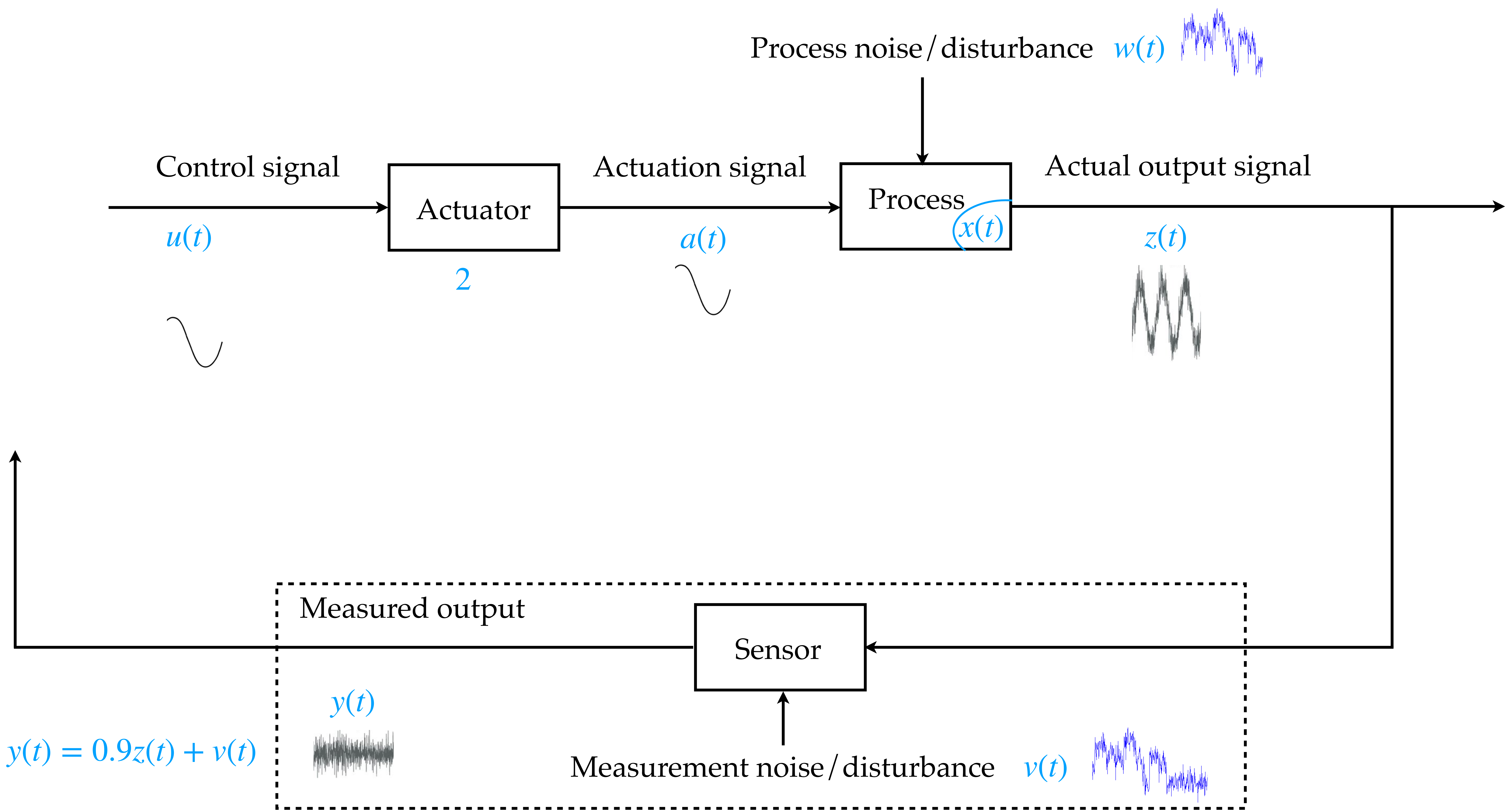
$$\begin{pmatrix} x_1(t+1) & = & -3x_1(t) + x_2(t) + a(t) \\ x_2(t+1) & = & x_1(t) - x_2(t) + 0.9a(t) + w(t) \end{pmatrix}$$

$$z(t) = x_1(t) + x_2(t)$$

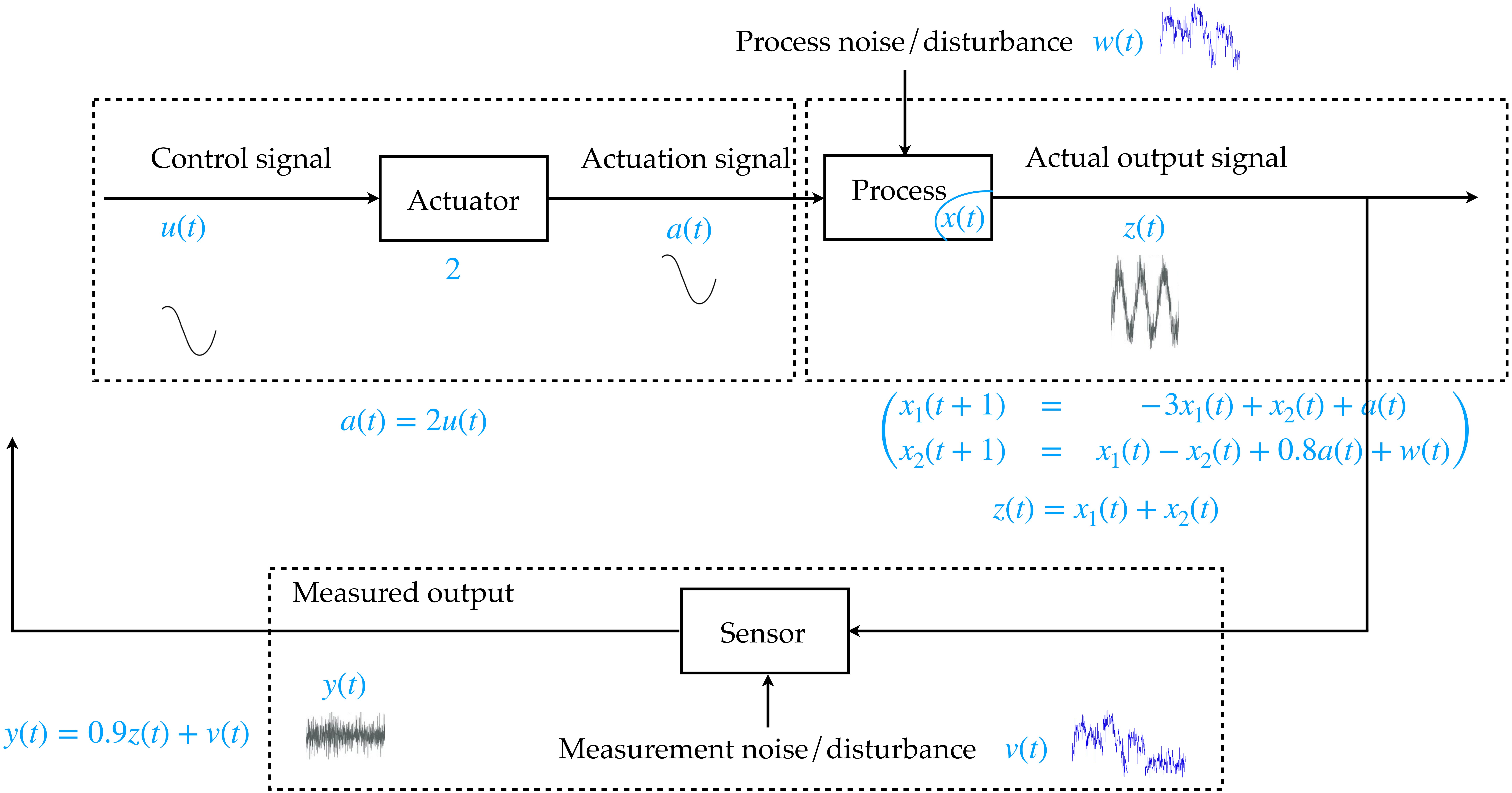
A discrete time example: process dynamics



A discrete time example: process dynamics



A discrete time example: process dynamics



A discrete time example: process dynamics

This is same as writing ….

$$\begin{pmatrix} x_1(t+1) & = & -3x_1(t) + x_2(t) + 2u(t) \\ x_2(t+1) & = & x_1(t) - x_2(t) + 1.6u(t) + w(t) \end{pmatrix}$$

$$y(t) = 0.9(x_1(t) + x_2(t)) + v(t)$$

Directly relates the control signal, measured output, process states and disturbances

Check this yourself!

A discrete time example: process dynamics

But what does the dynamics really mean here?

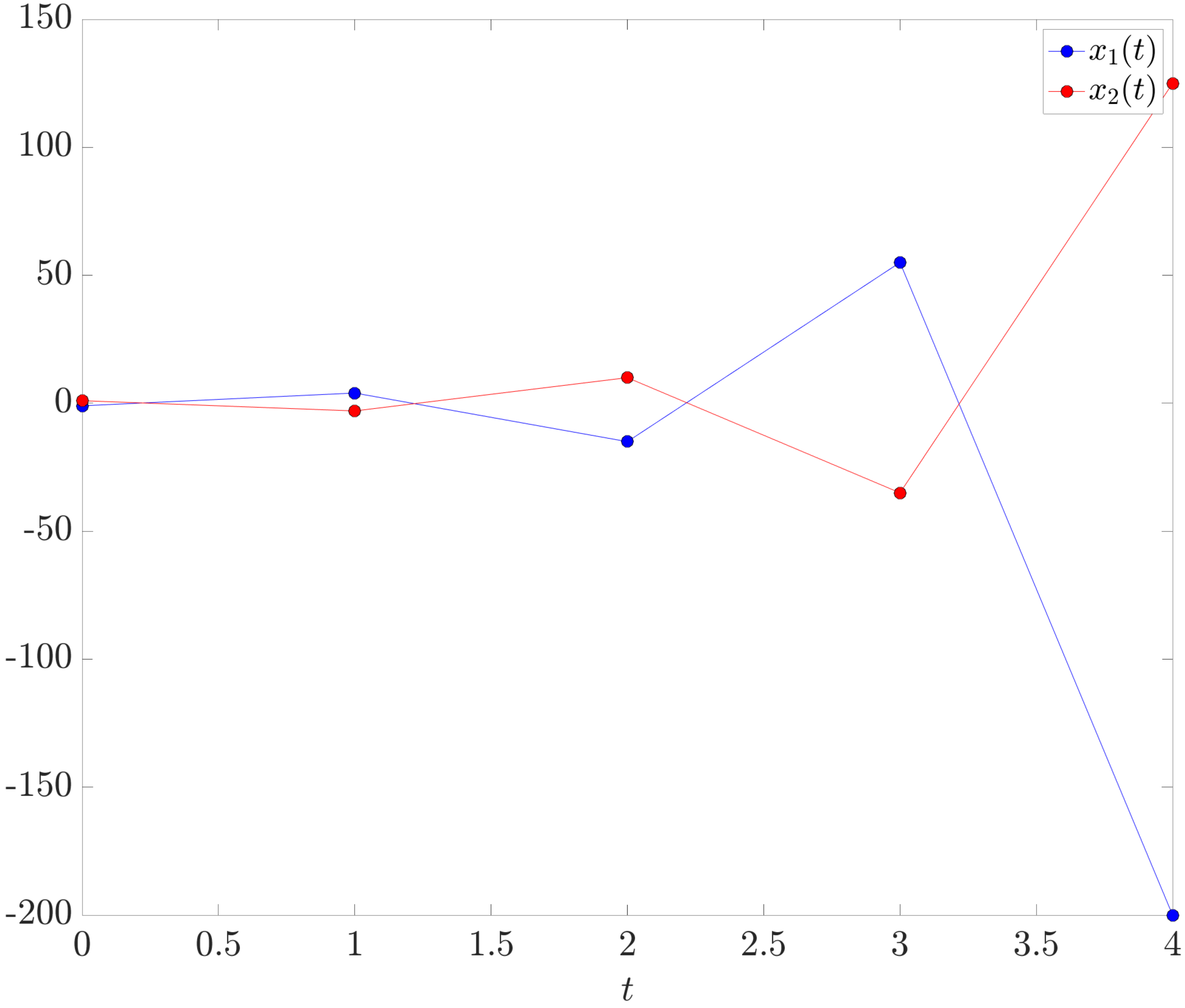
Suppose for simplicity, the control and the disturbances are zero. Then ….

$$\begin{pmatrix} x_1(t+1) & = & -3x_1(t) + x_2(t) \\ x_2(t+1) & = & x_1(t) - x_2(t) \end{pmatrix}$$

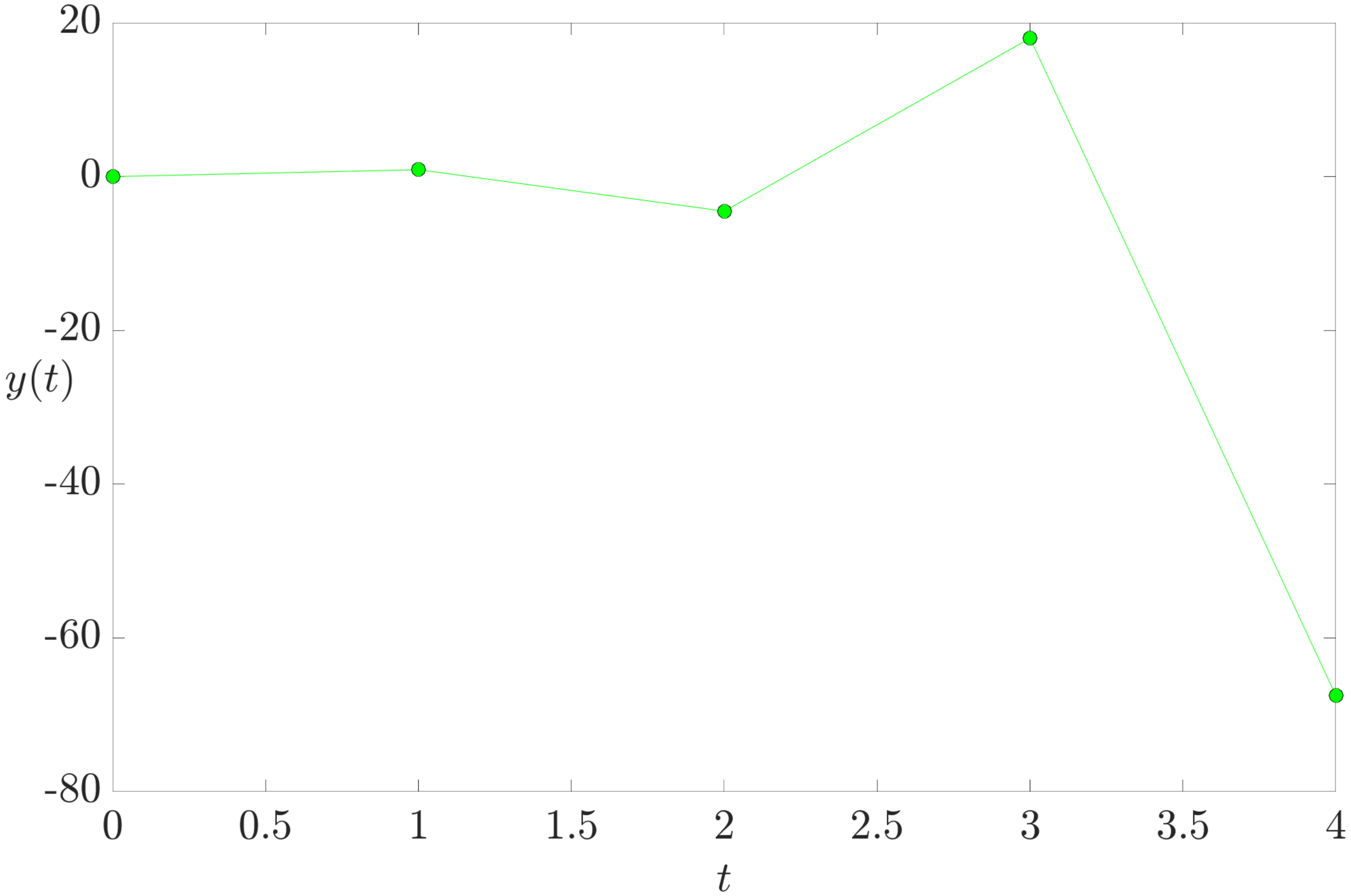
$$y(t) = 0.9(x_1(t) + x_2(t))$$

We can simulate this system with known initial states, say $\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

A discrete time example: process states



A discrete time example: measured output



In general, each box may have its own internal states