Stability and Stabilization

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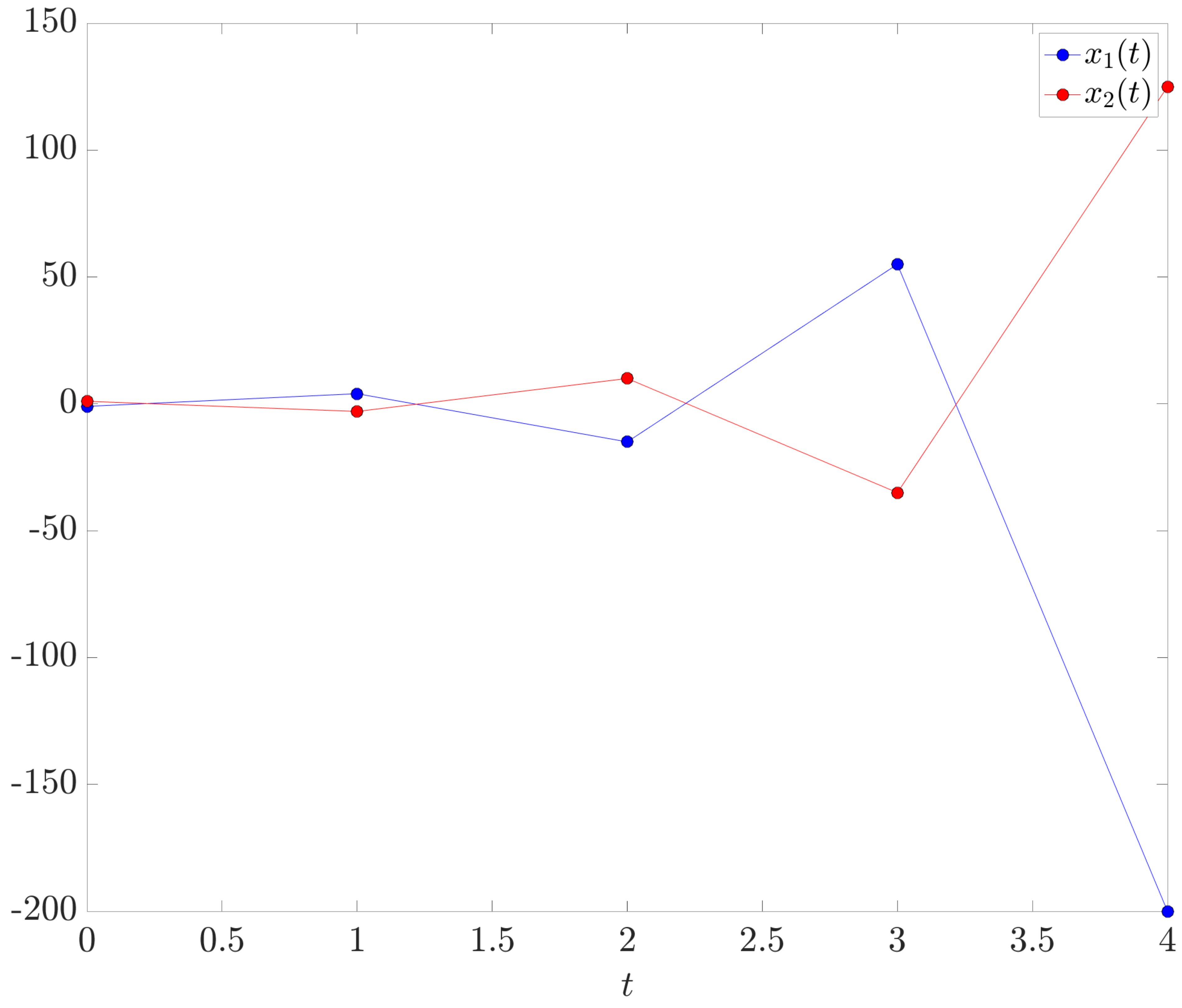
Recap: Dynamics in control systems

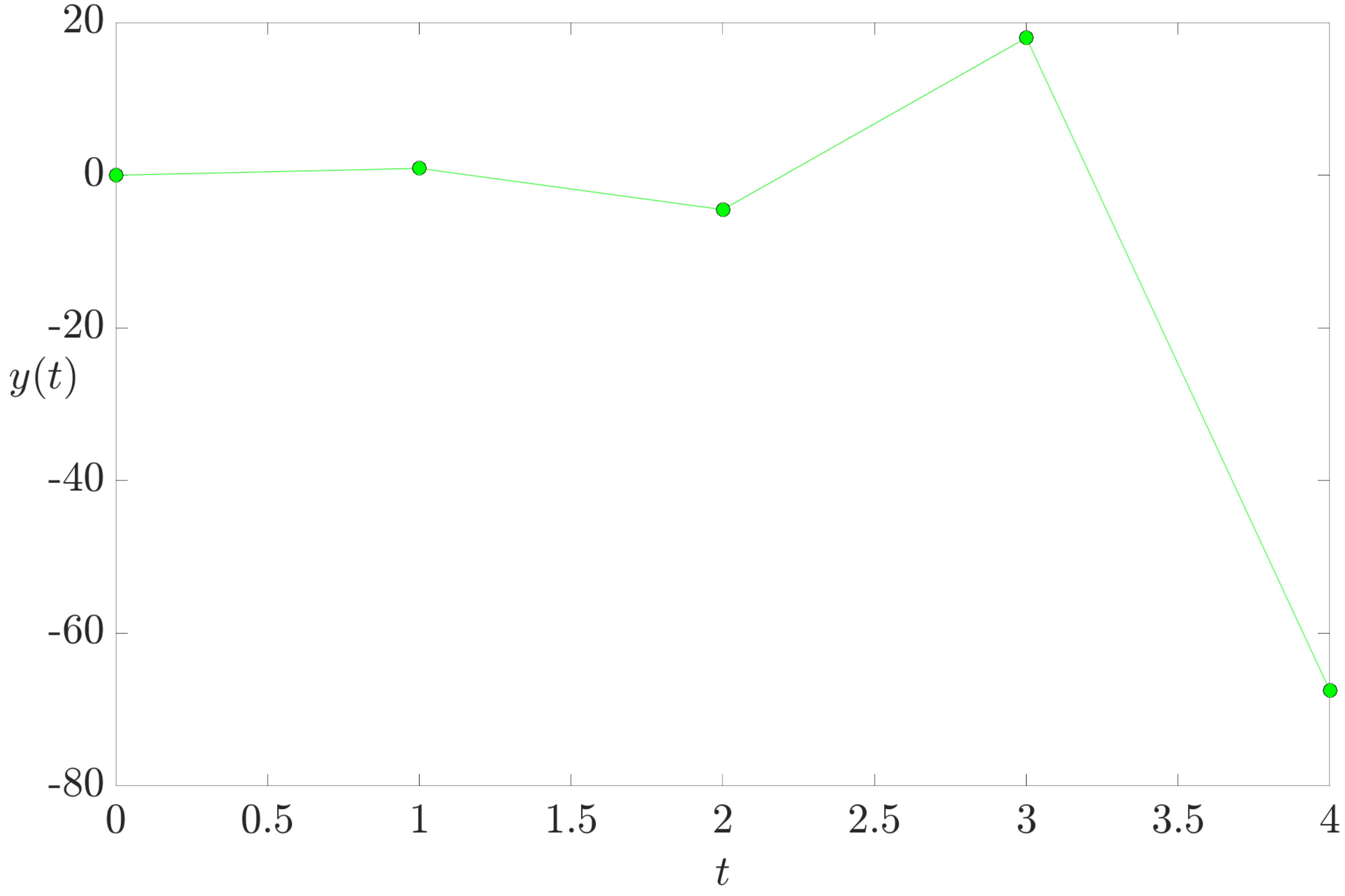
Recap: Discrete time dynamics example

Suppose for simplicity, the control and the disturbances are zero. Then substitute $u(t) \equiv 0$, $w(t) \equiv 0$, $v(t) \equiv 0$

$$\begin{pmatrix} x_1(t+1) & = & -3x_1(t) + x_2(t) + 2u(t) \\ x_2(t+1) & = & x_1(t) - x_2(t) + 1.6u(t) + w(t) \end{pmatrix}$$

$$y(t) = 0.9(x_1(t) + x_2(t)) + v(t)$$





Example: Discrete time dynamics of a wheeled mobile robot

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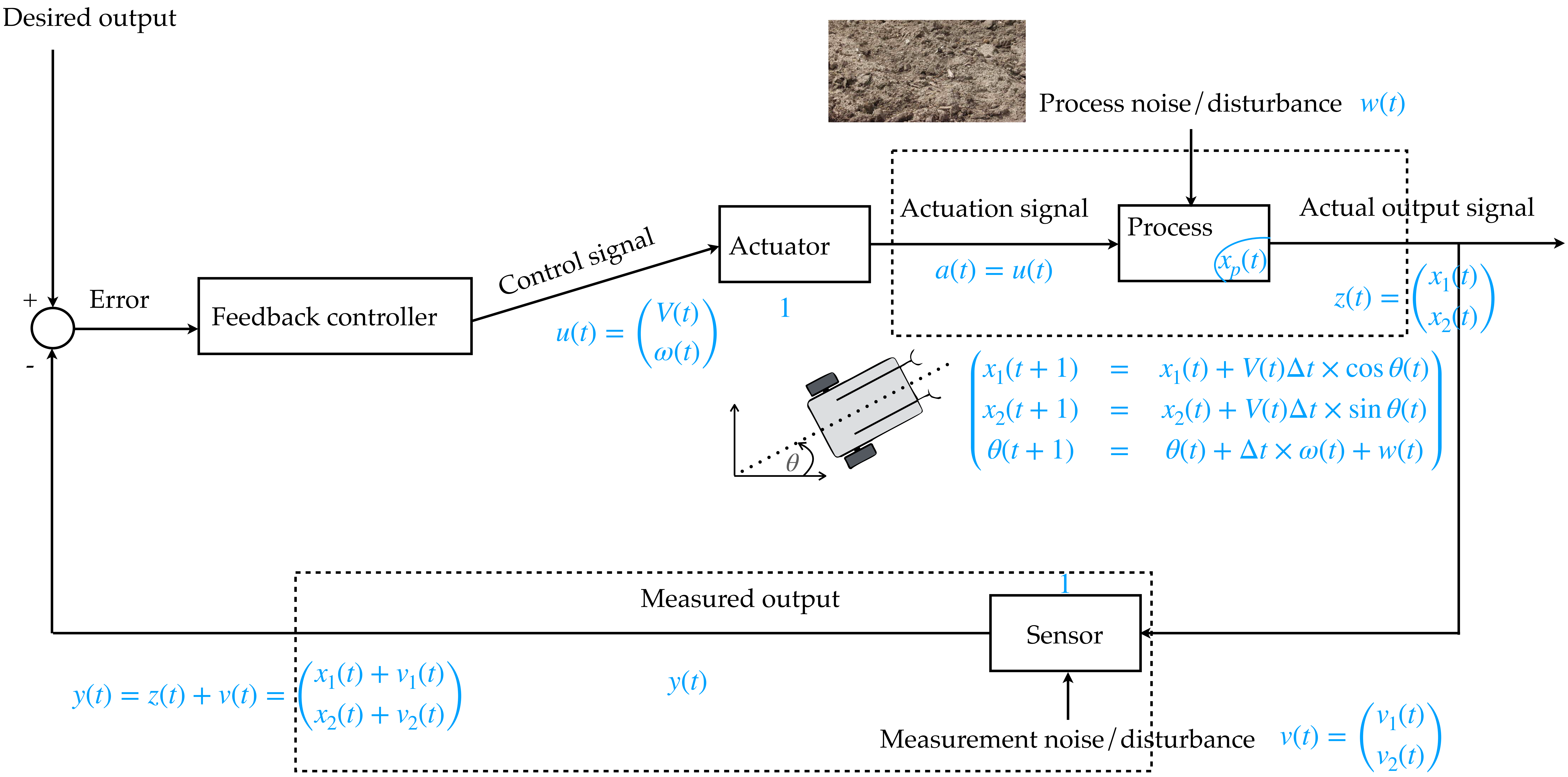
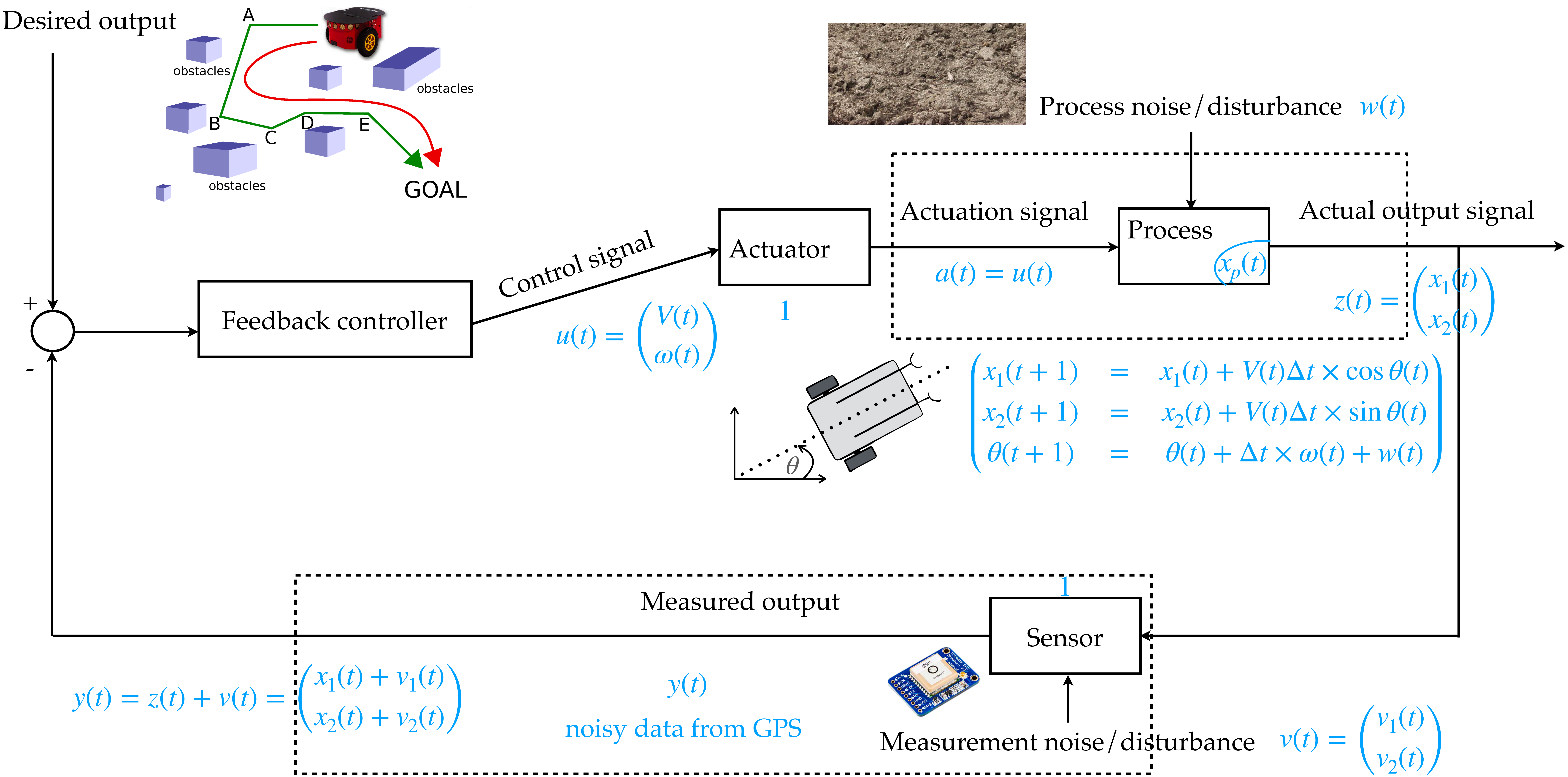


Example: Discrete time dynamics of a wheeled mobile robot



Instead of output, feedback may act on estimated/filtered state

Process dynamics may be “stable” or “unstable “

Process state is “stable” (S) about a point if

Start close to a point ⇝ Stay **arbitrarily** close to that point at all times

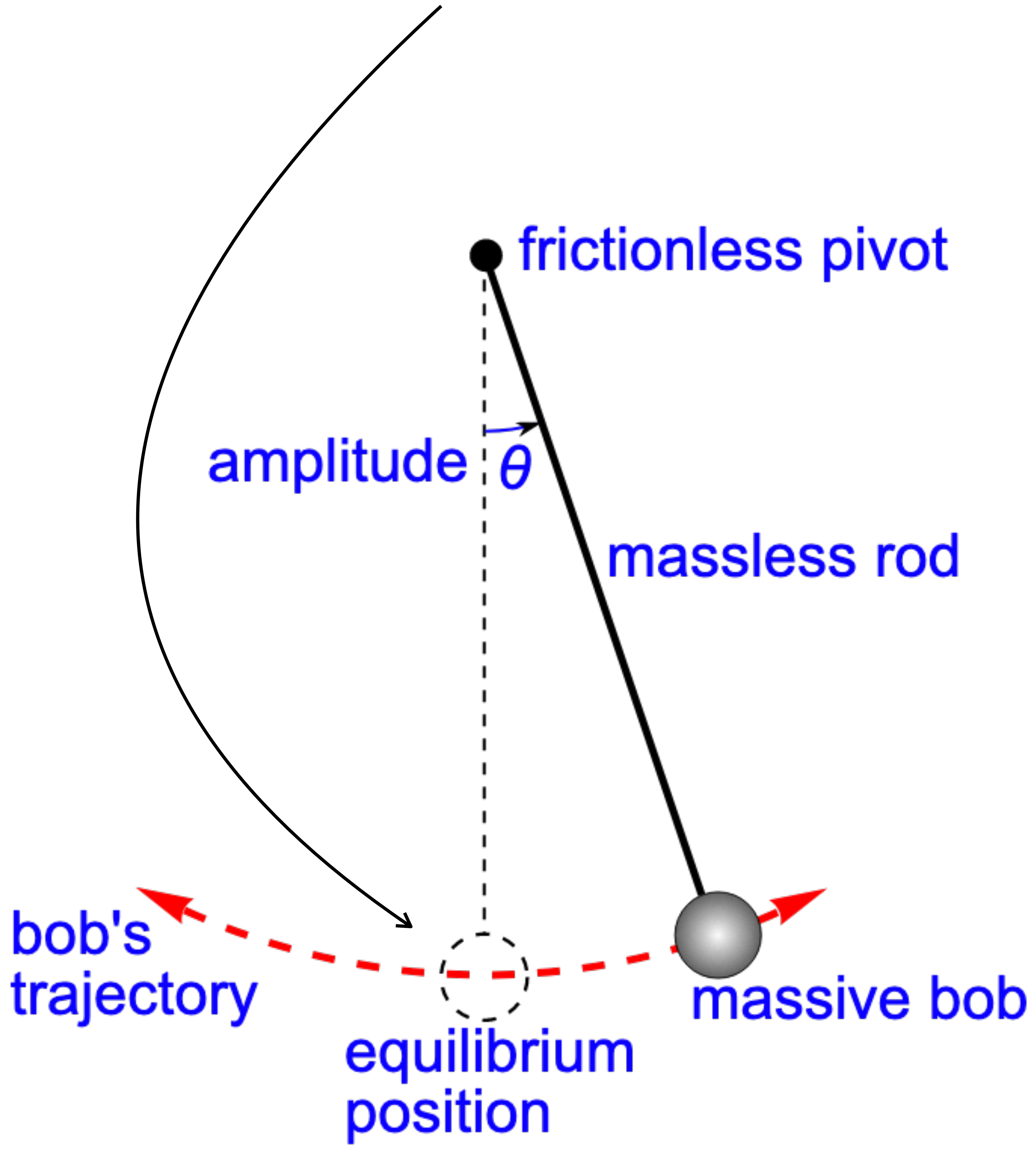
Process state is “asymptotically stable” (AS) about a point if

Start close to a point ⇝ **Settle** to that point if you wait long enough

Process state is “globally asymptotically stable” (GAS) about a point if

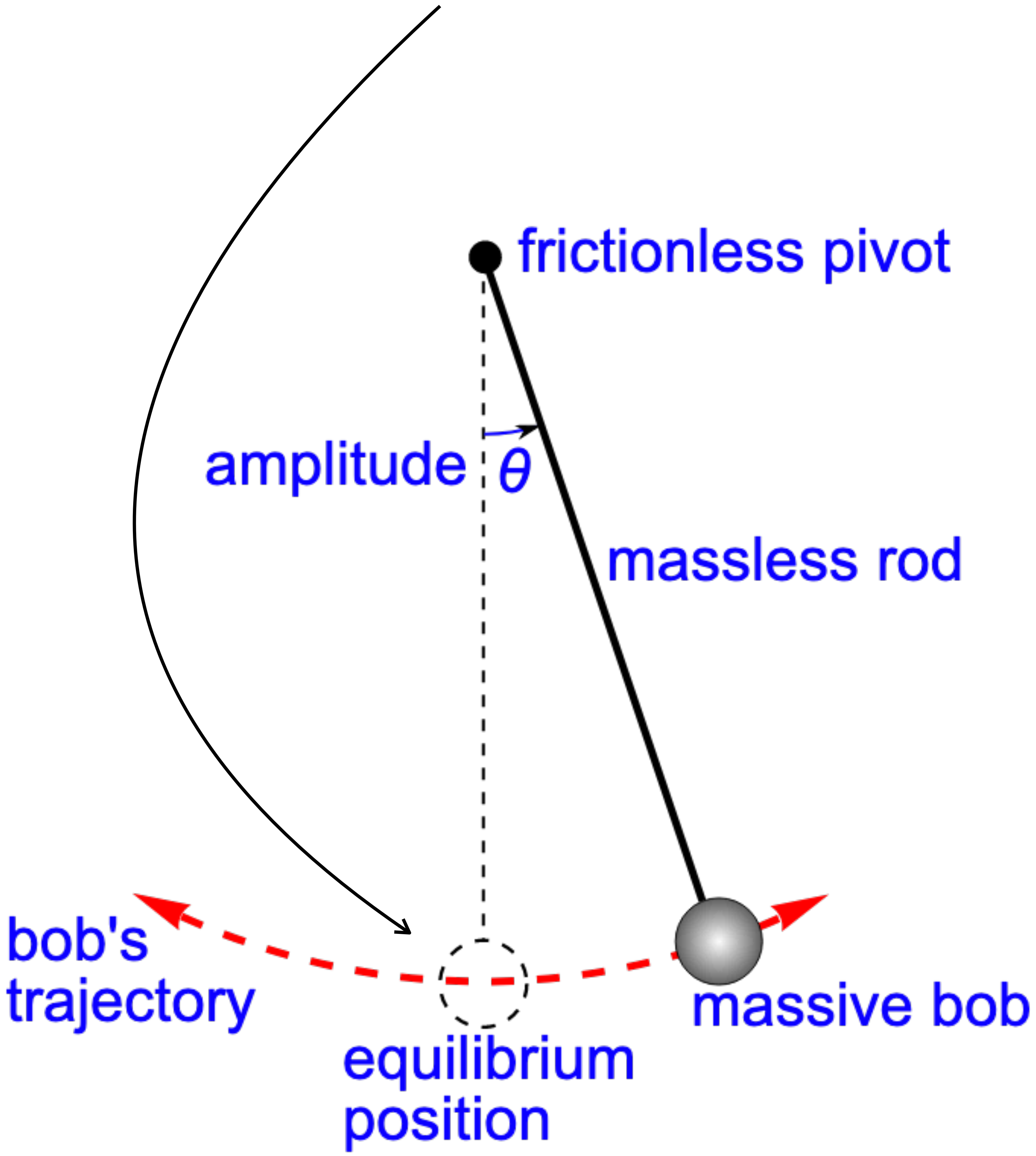
Start anywhere ⇝ **Settle** to that point if you wait long enough

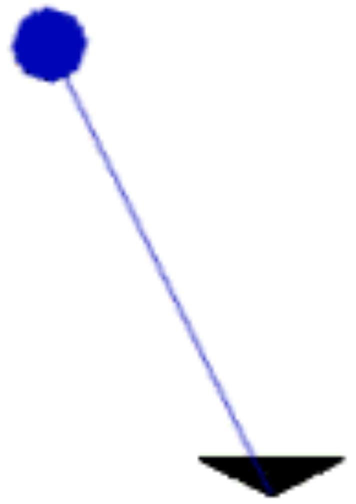
Is $(\theta, \omega) = (0{,}0)$ for simple pendulum dynamics S? AS? GAS?

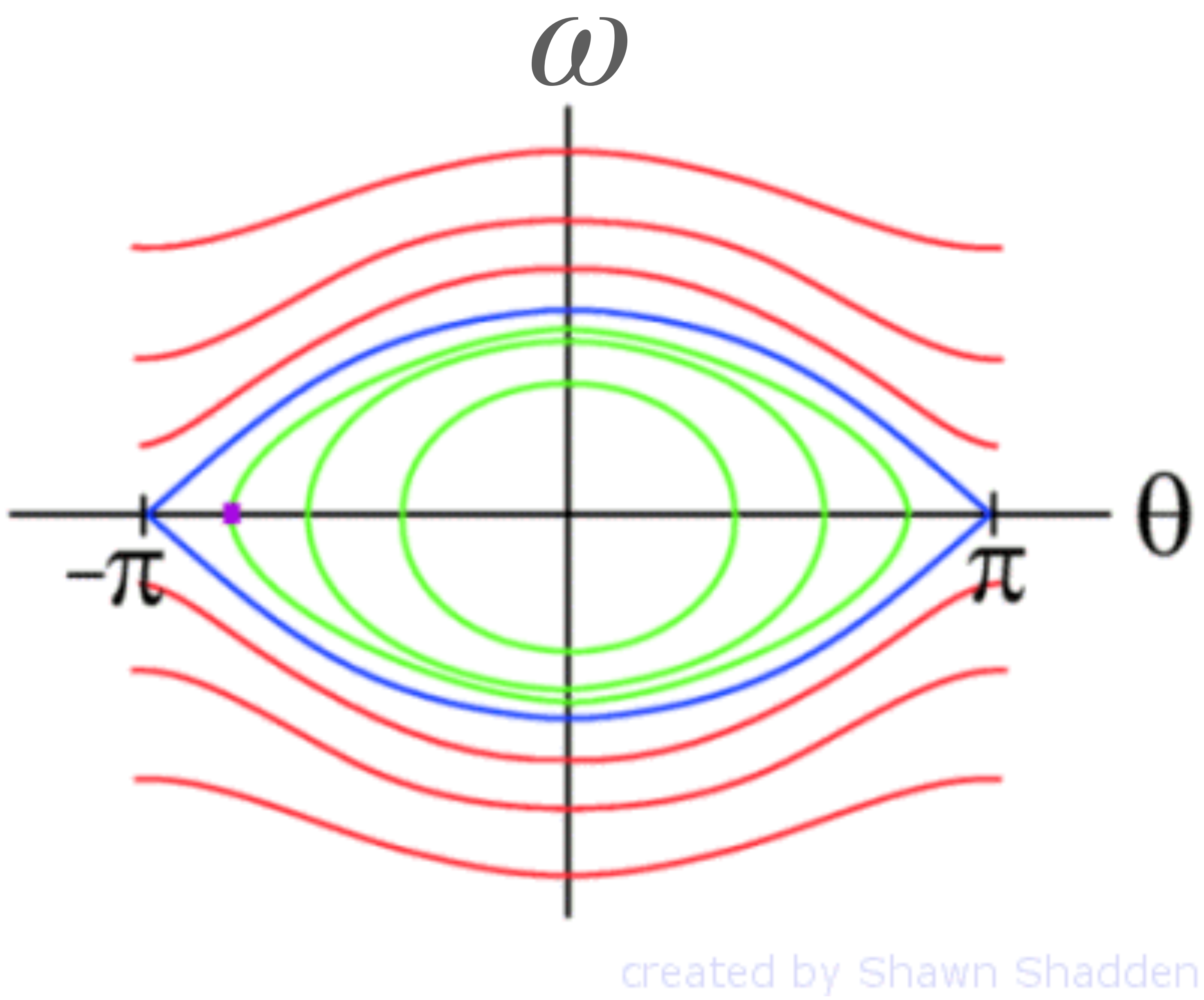


Simple pendulum

Is $(\theta, \omega) = (0,0)$ for simple pendulum dynamics S? AS? GAS?







Simple pendulum

Image credit: Shawn Shadden

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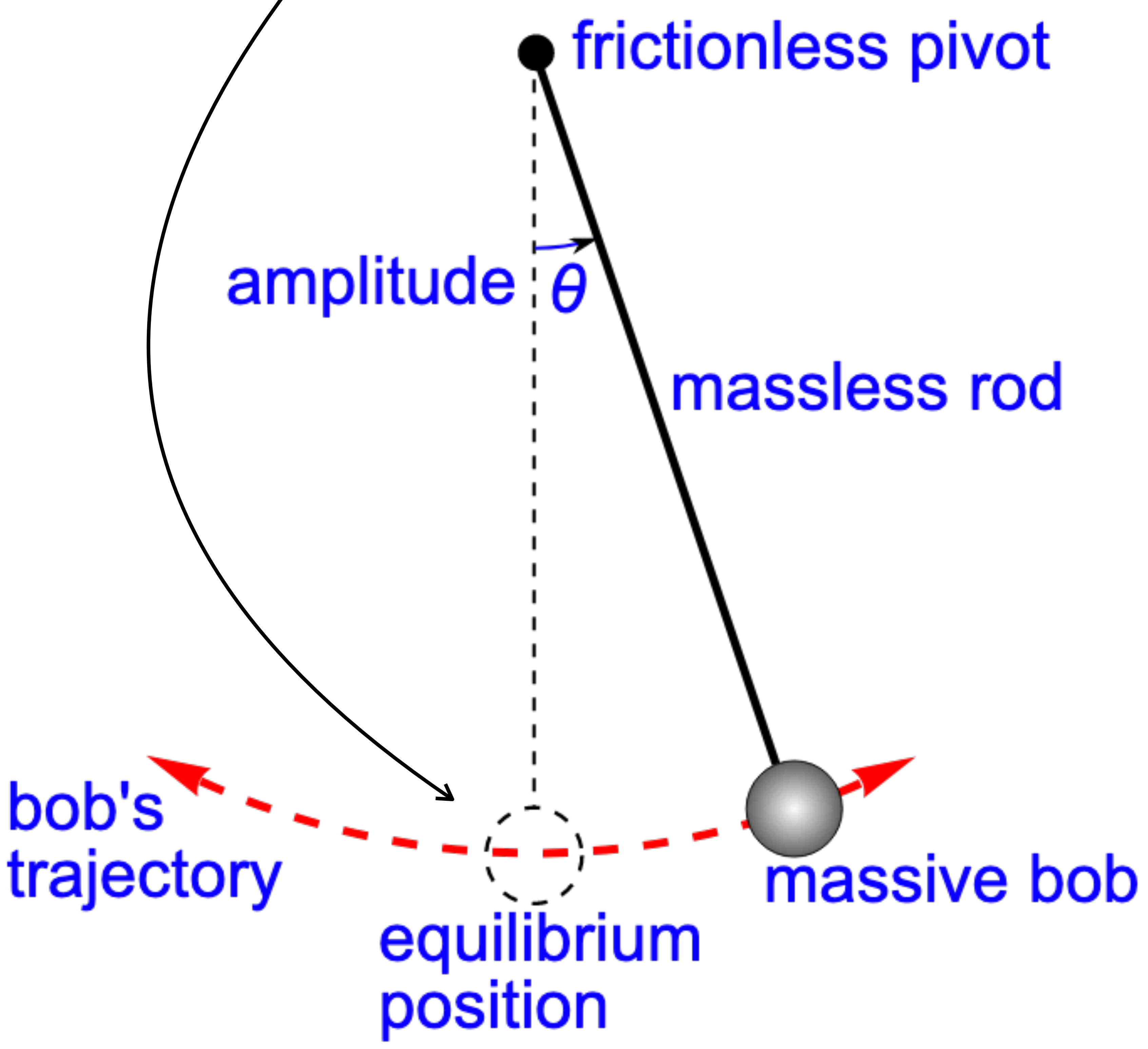


Simple pendulum

In vacuum $\rightsquigarrow$ (0,0) is S but not AS

In air $\rightsquigarrow$ (0,0) is S and AS, but **not GAS**

Why?

The point $(\theta, \omega) = (\pi, 0)$ for simple pendulum dynamics is …

Unstable equilibrium

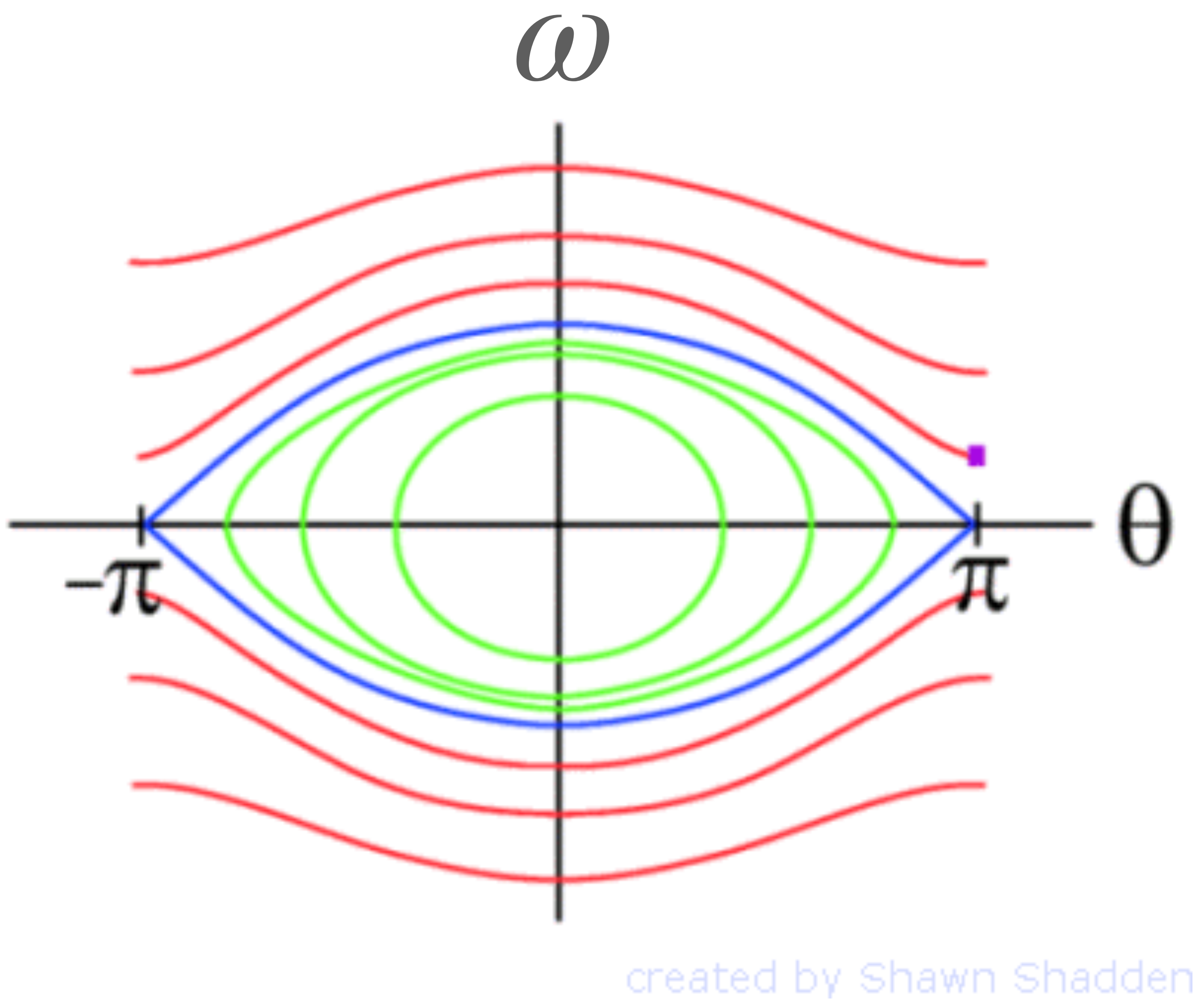
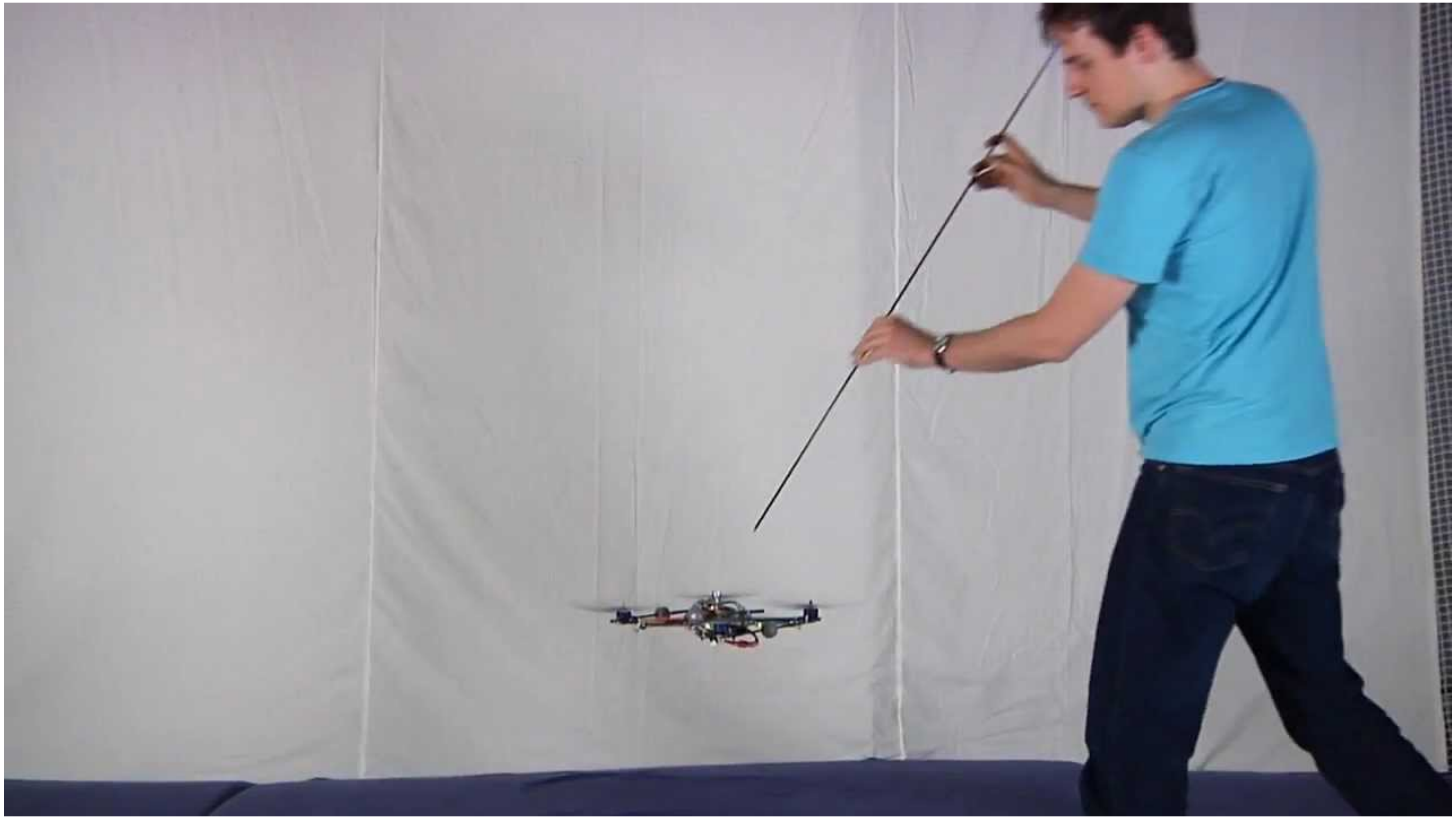


Image credit: Shawn Shadden

But we can use control to stabilize a process at an unstable point



Credit: Markus Hehn and Raffaello D'Andrea

But why bother about stabilization in engineering?
Why not directly engineer a process to be stable?

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Stable

Unstable but controller-in-the-loop stabilizable

But why bother about stabilization in engineering?
Why not directly engineer a process to be stable?

Hovering is unstable but controller-in-the-loop stabilizable

But why bother about stabilization in engineering? Why not directly engineer a process to be stable?

Pterosaurs: wingspan ~ 30 ft

Birds now

Flight is stable by design

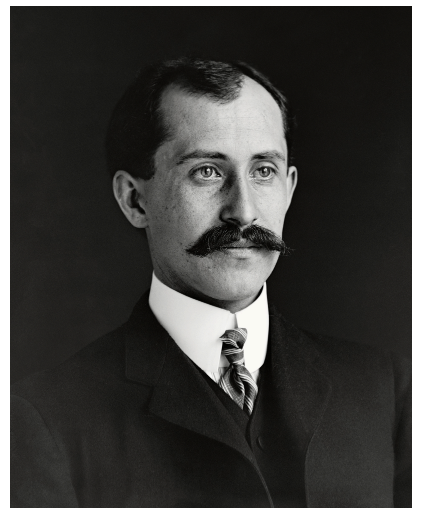
Unstable but controller-in-the-loop stabilizable

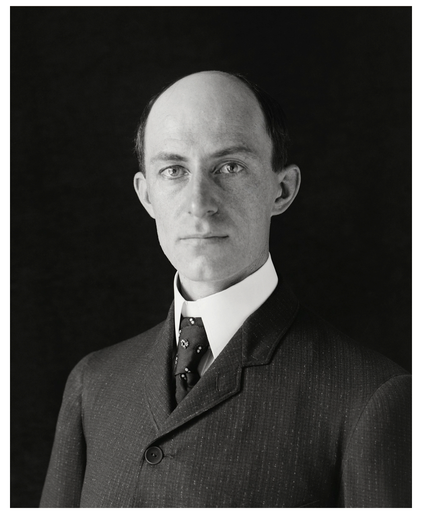
Inventing manned flight: heavier than air flying machines

"The Wright Brothers rejected the principle that aircraft should be made inherently so stable that the human pilot would only have to steer the vehicle, playing no part in stabilization. Instead they deliberately made their airplane with negative stability and depended on the human pilot to operate the movable surface controls so that the flying system - pilot and machine - would be stable. This resulted in an increase in maneuverability and controllability".

— Charles Stark Draper
Lecture at Royal Aeronautical Society, May 19, 1955

First flight of the Wright flyer on 17 December, 1903





Orville (left) and Wilbur Wright

Sperry's autopilot demo in Paris, 1912