AerE 5740: Optimal Control: Spring 2025

Iowa State University

Quiz 1: Background on OPT, (non)convex, positive (semi-)definite matrices

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

Fix integer $n \geq 2$. The set $\{\boldsymbol{X} \in \mathbb{R}^{n\times n} \mid X_{ij} \geq 0, \sum_{i=1}^{n} X_{ij} = 1, \sum_{j=1}^{n} X_{ij} = 1\}$

(A) is convex. `Reason:` Take any $\boldsymbol{X}_1, \boldsymbol{X}_2 \in \mathcal{X}$. Their average $\theta\boldsymbol{X}_1 + (1-\theta)\boldsymbol{X}_2 \in \mathcal{X}$ $\forall\theta \in [0,1]$.

(B) is nonconvex.

(C) could be convex or nonconvex depending on the value of integer $n \geq 2$.

Problem 2

For any nonzero vector $\boldsymbol{a} \in \mathbb{R}^n$, the matrix $\boldsymbol{A} = \boldsymbol{a}\boldsymbol{a}^\top$ is

(A) positive semidefinite. `Reason:` $\boldsymbol{x}^\top\boldsymbol{a}\boldsymbol{a}^\top\boldsymbol{x} = \left(\boldsymbol{a}^\top\boldsymbol{x}\right)^2 \geq 0 \; \forall\boldsymbol{x} \neq \boldsymbol{0}$. Strict inequality fails for $\boldsymbol{x}$ orthogonal to $\boldsymbol{a}$, so cannot guarantee positive definiteness.

(B) positive definite.

(C) symmetric but nothing more can be said about its definiteness.

Problem 3

The OPT problem

$$\min_{(x_1,x_2)\in\{(x_1,x_2)\in\mathbb{R}^2 | x_1^2+x_2^2=1\}} \quad x_1 - x_2$$

is nonconvex because its

(A) objective function is nonconvex.

(B) feasible set is nonconvex. `Reason:` feasible set is a circle (thus nonconvex). The objective is linear (thus convex).

(C) objective function and feasible set are both nonconvex.

Problem 4

If $\boldsymbol{A} \in \mathbb{S}^n$ has a negative entry somewhere along its main diagonal, then $\boldsymbol{A}$

(A) must be positive semidefinite.

(B) may or may not be positive semidefinite.

(C) cannot be positive semidefinite. `Reason:` All principal minors ≥ 0 check includes main diagonals, see example in Lec. 2, p. 13 top. `Alternative:` argue from definition: diagonal entries of $\boldsymbol{A}$ are $\boldsymbol{e}_i^\top \boldsymbol{A} \boldsymbol{e}_i$ where $\boldsymbol{e}_i$ is the ith standard basis vector.

Problem 5

A diagonal matrix is positive definite if and only if all entries in its main diagonal are

(A) positive. `Reason:` here the diagonal entries are the eigenvalues.

(B) nonnegative.

(C) real.