AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 10: Geometry of PMP, regulation vs. tracking, LQR formulation

Name:_____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

We say a dynamics $\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z})$ is volume-preserving if the divergence of the vector field is zero, that is, $\nabla \cdot \boldsymbol{f} = \sum_i \frac{\partial f_i}{\partial z_i} = 0$.
On the cotangent bundle, the OCP solution dynamics

$$\begin{pmatrix} \dot{\boldsymbol{x}}^{\text{opt}} \\ \dot{\boldsymbol{\lambda}}^{\text{opt}} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial H}{\partial \boldsymbol{\lambda}^{\text{opt}}} \\ -\dfrac{\partial H}{\partial \boldsymbol{x}^{\text{opt}}} \end{pmatrix}$$

(A) is volume preserving. `Reason:` here divergence $= \sum_i \frac{\partial^2 H}{\partial x_i^{\text{opt}} \lambda_i^{\text{opt}}} - \frac{\partial^2 H}{\partial \lambda_i^{\text{opt}} x_i^{\text{opt}}} = 0$.

(B) is NOT volume preserving.

(C) may or may not be volume preserving depending on the Hamiltonian H.

Problem 2

A rescue helicopter is trying to evacuate people from a wildfire location with strong wind gusts. To make the helicopter hover at constant altitude, the pilot engages an automatic "altitude-hold" controller. This controller is an example of

(A) regulator. `Reason:` Lec. 11 p. 13.

(B) path tracker.

(C) neither regulator nor tracker.

Problem 3

In the LQR formulation, the terminal cost weight matrix $\boldsymbol{M}$ must be

(A) symmetric, possibly sign-indefinite.

(B) orthogonal.

(C) positive semi-definite. `Reason:` Lec. 11 p. 15.

Problem 4

In the LQR formulation, the state cost-to-go weight matrix $\boldsymbol{Q}(t)$ must be

(A) positive semi-definite for ALL $t \in [0, T]$. `Reason:` Lec. 11 p. 15.

(B) positive semi-definite for SOME $t \in [0, T]$.

(C) positive semi-definite for NO $t \in [0, T]$.

Problem 5

In the LQR formulation, the control cost-to-go weight matrix $\boldsymbol{R}(t)$ must be

(A) positive definite for ALL $t \in [0, T]$. `Reason:` Lec. 11 p. 15.

(B) positive definite for SOME $t \in [0, T]$.

(C) positive definite for NO $t \in [0, T]$.