AerE 5740: Optimal Control: Spring 2025

Iowa State University

Quiz 15: Singular OCP, bang-bang control

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

All problems below are for the minimum time LTI OCP (so free final time T):

$$\begin{aligned}
\min_{\boldsymbol{u}} \quad & \int_0^T \mathrm{d}t \qquad\qquad (\star)\\
\text{subject to} \quad & \dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \boldsymbol{u},\\
& \boldsymbol{x}(t=0) = \boldsymbol{x}_0 \text{ given}, \quad \boldsymbol{x}(t=T) = \boldsymbol{x}_T \text{ given } \neq \boldsymbol{x}_0,\\
& \boldsymbol{u} \in [-1,1]^2,
\end{aligned}$$

where $\alpha \in \mathbb{R}$ is a fixed parameter.

Problem 1

The components of the optimal control $\boldsymbol{u}^{\text{opt}}(t)$ for $(\star)$, will have finitely many switchings between -1 and $+1$

(A) only for $\alpha \neq 0$.

(B) only for $\alpha \neq -1$.

(C) for any $\alpha \in \mathbb{R}$. `Reason:` Finitely many switching for such problems are guaranteed for any LTI system. See Lec. 16 p. 9.

Problem 2

Which of the following is TRUE for the optimal control $\boldsymbol{u}^{\text{opt}}(t) = (u_1^{\text{opt}}(t), u_2^{\text{opt}}(t))^\top$ for $(\star)$?

(A) $u_2^{\text{opt}}(t)$ is bang-bang for any $\alpha \in \mathbb{R}$. `Reason:` $\det[\boldsymbol{b}_2 | \boldsymbol{A}\boldsymbol{b}_2] = \det \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \neq 0$. See Lec. 16 p. 11.

(B) $u_2^{\text{opt}}(t)$ is NOT bang-bang for any $\alpha \in \mathbb{R}$.

(C) $u_2^{\text{opt}}(t)$ is bang-bang only for $\alpha \neq -1$.

Problem 3

Which of the following is TRUE for the optimal control $\boldsymbol{u}^{\text{opt}}(t) = (u_1^{\text{opt}}(t), u_2^{\text{opt}}(t))^\top$ for $(\star)$?

(A) BOTH $u_1^{\text{opt}}(t), u_2^{\text{opt}}(t)$ are bang-bang for any $\alpha \in \mathbb{R}$.

(B) BOTH $u_1^{\text{opt}}(t), u_2^{\text{opt}}(t)$ are bang-bang for any $\alpha \neq -1$. `Reason:` $\det[\boldsymbol{b}_1|\boldsymbol{A}\boldsymbol{b}_1] = \det \begin{bmatrix} \alpha & \alpha+3 \\ 1 & -2 \end{bmatrix} = -2\alpha - (\alpha+3) \neq 0$ if and only if $\alpha \neq -1$. See Lec. 16 p. 10-11.

(C) BOTH $u_1^{\text{opt}}(t), u_2^{\text{opt}}(t)$ are bang-bang for any $\alpha \neq 1.5$.

Problem 4

In $(\star)$, BOTH $u_1^{\text{opt}}(t), u_2^{\text{opt}}(t)$ can have <u>at most one switching</u> provided

(A) $\alpha \neq 1.5$.

(B) $\alpha \neq -1$ `Reason:` eigenvalues of given triangular $\boldsymbol{A}$ matrix are its real diagonal entries: $1, -2$. See Lec. 16 p. 11.

(C) any $\alpha \in \mathbb{R}$.

Problem 5

In $(\star)$, if we <u>change the state matrix</u> from $\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$ to $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$, then <u>answers to which of the Problems 1-4 will change</u>?

(A) All of them.

(B) None of them. `Reason:` modified matrix $\boldsymbol{A}$ will still have all its eigenvalues real: $1, 2$. See Lec. 16 p. 11.

(C) Problem 4 only.