AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 18: Discrete Time Finite Horizon DP

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Given 1D controlled dynamical system

$$x_{k+1} = x_k + u_k, \quad \text{control } u_k \in \{-1, 0\}, \quad \text{time index } k = 0, 1, \quad \text{initial condition } x_0 = 1.$$

Consider the problem

$$\underset{\gamma\in\Gamma}{\text{minimize}} \quad \alpha\left(x_1^2 + x_2^2\right) + (1-\alpha)\left(u_0^2 + u_1^2\right)$$

subject to the above dynamics, where Γ is the collection of all history dependent randomized policies, and the known constant α satisfies $0 \leq \alpha \leq 1$.

Problem 1

Let $\mathcal{X}_k$ denote the state space for each $k = 0, 1, 2$. From the initial condition, $\mathcal{X}_0 = \{1\}$.

(A) $\mathcal{X}_1 = \{-1, 1\}$, $\mathcal{X}_2 = \{0, 1\}$.

(B) $\mathcal{X}_1 = \{0, 1\}$, $\mathcal{X}_2 = \{-1, 1\}$.

(C) $\mathcal{X}_1 = \{0, 1\}$, $\mathcal{X}_2 = \{-1, 0, 1\}$. `Reason:` $u_k \in \{-1, 0\}$.

Problem 2

Re-writing the given objective in standard form: $c_2(x_2) + \sum_{k=0}^{2-1} c_k(x_k, u_k)$, we identify $c_2(x_2) = \alpha x_2^2$, and

(A) $c_0(x_0, u_0) = \alpha u_0^2$, $c_1(x_1, u_1) = \alpha x_1^2 + (1-\alpha)u_1^2$.

(B) $c_0(x_0, u_0) = (1-\alpha)u_0^2$, $c_1(x_1, u_1) = \alpha x_1^2 + (1-\alpha)u_1^2$. `Reason:` direct identification.

(B) $c_0(x_0, u_0) = (1-\alpha)u_0^2$, $c_1(x_1, u_1) = (1-\alpha)x_1^2 + \alpha u_1^2$.

Problem 3

To perform DP recursion, we start with $V_2(x_2) = c_2(x_2) = \alpha x_2^2$. Then the DP equation

$$V_1(x_1) = \underset{u \in \{-1,0\}}{\text{minimum}} \ (c_1(x_1, u) + V_2(x_1 + u))$$

gives (recall $x_1 \in \mathcal{X}_1$)

(A) $V_1(0) = 0$ and $V_1(1) = 1$.

(B) $V_1(0) = 0$ and $V_1(1) = \text{minimum}\{\alpha, 1\}$.

(C) $V_1(0) = 0$ and $V_1(1) = \text{minimum}\{2\alpha, 1\}$. `Reason:` substituting c_1 and V_2 in the given DP equation gives $V_1(x_1) = \text{minimum}\{2\alpha x_1^2, 2\alpha x_1^2 - 2\alpha x_1 + 1\}$. Then use $x_1 \in \mathcal{X}_1 = \{0, 1\}$.

Problem 4

Recall that $x_0 \in \mathcal{X}_0 = \{1\}$. The DP equation

$$V_0(x_0) = \underset{u \in \{-1,0\}}{\text{minimum}} \ (c_0(x_0, u) + V_1(x_0 + u))$$

gives

(A) $V_0(1) = \text{minimum}\{1 - \alpha, \text{minimum}\{2\alpha, 1\}\}$. `Reason:` substitute c_0 and V_1 in the given DP equation.

(B) $V_0(1) = \text{minimum}\{1 - \alpha, \text{minimum}\{3\alpha, 1\}\}$.

(C) $V_0(1) = \text{minimum}\{\alpha, \text{minimum}\{2\alpha, 1\}\}$.

Problem 5

Noticing that

$$\text{minimum}\{2\alpha, 1\} = \begin{cases} 2\alpha & \text{if} \quad \alpha \leq 1/2, \\ 1 & \text{if} \quad \alpha > 1/2, \end{cases}$$

we can simplify $V_0(1)$ to be equal to

(A) $\text{minimum}\{1 - \alpha, 2\alpha\}$. `Reason:`

$$\begin{aligned} V_0(1) &= \min\{1 - \alpha, \min\{2\alpha, 1\}\} \\ &= \begin{cases} \min\{1 - \alpha, 2\alpha\} & \text{if} \quad \alpha \leq 1/2, \\ \underbrace{\min\{1 - \alpha, 1\}}_{=1-\alpha} & \text{if} \quad \alpha > 1/2, \end{cases} \\ &= \min\{\min\{1 - \alpha, 2\alpha\}, 1 - \alpha\} \\ &= \min\{1 - \alpha, 2\alpha\}. \end{aligned}$$

(B) $\text{minimum}\{1 - \alpha, \alpha\}$.

(C) $1 - \alpha$.