AerE 5740: Optimal Control: Spring 2025

Iowa State University

Quiz 2: Background on LP, SDP, LTI control systems

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

In two dimensional plane, the first quadrant $\{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 \geq 0\}$ is

(A) a polyhedron because we can re-write the two inequalities $x_1 \geq 0, x_2 \geq 0$ as a single linear matrix-vector inequality $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. `Reason:` as stated in this option.

(B) a spectrahedron but not a polyhedron.

(C) a nonconvex set.

Problem 2

Consider the 2×2 symmetric matrix $\boldsymbol{X} = \begin{bmatrix} X_{11} & X_{12} \\ X_{12} & X_{22} \end{bmatrix}$. The OPT problem

$$\begin{aligned} \min_{\boldsymbol{X} \in \mathbb{S}^2_+} \quad & X_{11} + X_{22} \\ \text{subject to} \quad & X_{11} - X_{12} + X_{22} \leq 5, \end{aligned}$$

is a

(A) linear program.

(B) semidefinite program. `Reason:` This is in the standard SDP form given in Lec. 3 p. 6 with $n = 2, m = 1, \boldsymbol{C} = \boldsymbol{I}, \boldsymbol{A}_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, b_1 = 5$. Useful tip: $\langle \boldsymbol{A}, \boldsymbol{X} \rangle = \sum_{i,j} A_{ij} X_{ij}$.

(C) convex but not semidefinite program.

Problem 3

The continuous-time linear time invariant (LTI) control system $\ddot{x} = u$ has state matrix

(A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. `Reason:` let $x_1 := x, x_2 := \dot{x}$ and rewrite as first order vector ODE/state space form.

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Problem 4

If an LTI control system is controllable, then it

(A) must also be stabilizable. `Reason:` Lec. 3 p. 10 top.

(B) may or may not be stabilizable.

(C) cannot be stabilizable.

Problem 5

Which of the following feedback gain k stabilizes the 1-state-1-input discrete-time control system $x_{t+1} = x_t + u_t$ with control $u_t = -kx_t$?

(A) $k = 0$.

(B) $k = 0.5$. `Reason:` closed-loop dynamics $x_{t+1} = (1-k)x_t$, which is stable iff $|1-k| < 1$ or $0 < k < 2$.

(C) $k = 2$.