AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 21: Continuous Time Finite Horizon DP

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Consider the following (fixed terminal time $T < \infty$) deterministic OCP in continuous time:

$$\begin{aligned} \min_{\gamma \in \Gamma} \quad & (x(T) - 3)^4 \; + \int_0^T \frac{1}{2} u^2 \, \mathrm{d}t \\ \text{subject to} \quad & \ddot{x} = u, \end{aligned}$$

where Γ denotes the set of all history-dependent randomized policies. All five questions below are about the DP equation for this specific OCP.

In general, the DP equation for a deterministic OCP is the HJB PDE IVP of the form

$$\frac{\partial V}{\partial t} + \underset{\boldsymbol{u} \in \mathcal{U}}{\text{minimum}} \left\{ L(t, \boldsymbol{x}, \boldsymbol{u}) + \left\langle \frac{\partial V}{\partial \boldsymbol{x}}, \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) \right\rangle \right\} = 0, \qquad V(T, \boldsymbol{x}) = \phi(T, \boldsymbol{x}).$$

Problem 1

For the specific OCP, its Lagrangian L equals

(A) $\frac{1}{2} u^2$. `Reason:` given integrand for cost-to-go.

(B) $(x - 3)^4$.

(C) $(x - 3)^4 + \frac{1}{2} u^2$.

Problem 2

For the specific OCP, its HJB PDE IVP in unknown value function $V(t, x_1, x_2)$ is

(A) $\dfrac{\partial V}{\partial t} + x_2 \dfrac{\partial V}{\partial x_1} - \dfrac{1}{2} \left(\dfrac{\partial V}{\partial x_2} \right)^2 = 0, \quad V(T, x_1, x_2) = (x_1 - 3)^4$. `Reason:` substitute $L = \frac{1}{2} u^2$, $\boldsymbol{f} = \begin{pmatrix} x_2 \\ u \end{pmatrix}$, $\phi = (x_1 - 3)^4$ in the general HJB PDE IVP and simplify.

(B) $\dfrac{\partial V}{\partial t} + x_1 \dfrac{\partial V}{\partial x_2} - \dfrac{1}{2}\left(\dfrac{\partial V}{\partial x_2}\right)^2 = 0, \quad V(T, x_1, x_2) = (x_1 - 3)^4.$

(C) $\dfrac{\partial V}{\partial t} + x_2 \dfrac{\partial V}{\partial x_1} - \dfrac{1}{2}\left(\dfrac{\partial V}{\partial x_2}\right)^2 = 0, \quad V(T, x_1, x_2) = (x_2 - 3)^4.$

Problem 3

For the specific OCP, its HJB PDE is a

(A) first order linear PDE.

(B) first order nonlinear PDE. `Reason:` order of a PDE is the order of it highest derivative, here first derivative. Nonlinear in gradient of V. See also Lec. 22 p. 8.

(C) second order nonlinear PDE.

Problem 4

For the specific OCP, its optimal control u^{opt} can be recovered from the solution of the HJB PDE IVP as

(A) $u^{\text{opt}} = \dfrac{\partial V}{\partial x_2}$.

(B) $u^{\text{opt}} = -\dfrac{\partial V}{\partial x_2}$. `Reason:` minimizer in the HJB PDE.

(C) $u^{\text{opt}} = -\dfrac{\partial V}{\partial x_1}$.

Problem 5

For the specific OCP, its HJB PDE IVP cannot be solved using the Hopf-Lax formula

(A) because its Lagrangian or dynamics has explicit time dependence.

(B) because the optimized Hamiltonian has explicit state dependence. `Reason:` x_2 appears.

(C) because the optimized Hamiltonian is independent of the value function.