AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 23: Stochastic DP in Continuous Time

Name:_____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

The controlled relative motion of a satellite in geocentric orbit is given by the SDE

$$\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \\ \mathrm{d}v_x \\ \mathrm{d}v_y \\ \mathrm{d}v_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \\ -\dfrac{ax}{(x^2+y^2+z^2))^{3/2}} - bv_x + F_x \\ -\dfrac{ay}{(x^2+y^2+z^2))^{3/2}} - bv_y + F_y \\ -\dfrac{az}{(x^2+y^2+z^2))^{3/2}} - bv_z + F_z \end{pmatrix} \mathrm{d}t + c \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{d}w_1 \\ \mathrm{d}w_2 \\ \mathrm{d}w_3 \end{pmatrix},$$

where the state $\boldsymbol{x} := (x, y, z, v_x, v_y, v_z)^\top \in \mathbb{R}^6$, the control $\boldsymbol{u} := (F_x, F_y, F_z)^\top \in \mathbb{R}^3$, the process noise $\mathrm{d}\boldsymbol{w} = (\mathrm{d}w_1, \mathrm{d}w_2, \mathrm{d}w_3)^\top \in \mathbb{R}^3$, and the constant parameters $a, b, c > 0$. We can re-write this SDE in the standard vector form

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})\mathrm{d}t + \boldsymbol{\sigma}(t, \boldsymbol{x}, \boldsymbol{u})\mathrm{d}\boldsymbol{w},$$

where the diffusion coefficient $\boldsymbol{\sigma}$ is the 6×3 matrix given by

(A) $\begin{bmatrix} \boldsymbol{0}_{3\times3} \\ c\boldsymbol{I}_{3\times3} \end{bmatrix}$ where $\boldsymbol{0}, \boldsymbol{I}$ denote zero and identity matrices, respectively, of appropriate sizes. `Reason`: $\mathrm{d}\boldsymbol{w} \in \mathbb{R}^3$.

(B) $\begin{bmatrix} c\boldsymbol{I}_{3\times3} \\ \boldsymbol{0}_{3\times3} \end{bmatrix}$ where $\boldsymbol{0}, \boldsymbol{I}$ denote zero and identity matrices, respectively, of appropriate sizes.

(C) $c\boldsymbol{I}_{6\times6}$ where $\boldsymbol{I}$ denotes identity matrix of appropriate size.

Problem 2

In any controlled SDE $\mathrm{d}\boldsymbol{x} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})\mathrm{d}t + \boldsymbol{\sigma}(t, \boldsymbol{x}, \boldsymbol{u})\mathrm{d}\boldsymbol{w}$, the diffusion tensor $\boldsymbol{\sigma}\boldsymbol{\sigma}^\top$ is

(A) symmetric but could be sign indefinite.

(B) symmetric positive semidefinite irrespective of the size of $\boldsymbol{\sigma}$. `Reason`: Lec. 24 p. 9.

(C) symmetric positive semidefinite only if $\boldsymbol{\sigma}$ is a square matrix.

Problem 3

Consider the function $V(x_1, x_2) = x_1 - x_2^2$. Its Hessian Hess$V$ equals

(A) $1 - 2x_2$.

(B) $\begin{pmatrix} x_1 \\ -2x_2 \end{pmatrix}$.

(C) $\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}$. `Reason`: Hessian of a bivariate function must be 2×2 symmetric matrix.

Problem 4

The HJB PDE for continuous time stochastic DP

$$\frac{\partial V}{\partial t} + \underset{\boldsymbol{u} \in \mathcal{U}}{\text{minimum}} \left\{ L + \langle \nabla_{\boldsymbol{x}} V, \boldsymbol{f} \rangle + \frac{1}{2} \text{trace} \left(\boldsymbol{\sigma} \boldsymbol{\sigma}^\top \text{Hess} V \right) \right\} = 0,$$

is a

(A) first order nonlinear PDE.

(B) second order linear PDE.

(C) second order nonlinear PDE. `Reason`: Lec. 24 p. 7.

Problem 5

In the continuous time stochastic DP objective, the operator $\mathbb{E}$ stands for

(A) expected value. `Reason`: the objective is average cost.

(B) exponential.

(C) elephant.