AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 24: Second Order HJB PDE IVP

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

For fully observed continuous time finite horizon LQR and LQG problems with the same problem data, the corresponding optimal controllers are

(A) identical. `Reason`: Lec. 25 p. 4.

(B) different.

(C) identical or different depending on the diffusion coefficient.

Problem 2

For fully observed continuous time finite horizon LQR and LQG problems with the same problem data, let their optimal costs be J_{LQR} and J_{LQG}. Then

(A) $J_{\mathrm{LQR}} = J_{\mathrm{LQG}}$.

(B) $J_{\mathrm{LQR}} > J_{\mathrm{LQG}}$.

(C) $J_{\mathrm{LQR}} < J_{\mathrm{LQG}}$. `Reason`: Lec. 25 p. 1, 4. Notice also from Lec. 25 p. 3 that $v(t) = \int_t^T \frac{1}{2}\mathrm{trace}(\sigma\sigma^\top P(\tau))\mathrm{d}\tau \geq 0$ because $\mathrm{trace}(XY) = \mathrm{trace}(X\sqrt{Y}\sqrt{Y}) = \mathrm{trace}(\sqrt{Y}X\sqrt{Y}) \geq 0$ whenever $X, Y \succeq 0$.

Problem 3

The fully observed continuous time finite horizon LQG problem has a value function of the form $V(t,\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{P}(t)\boldsymbol{x} + v(t)$. Here $\boldsymbol{P}(t)$ solves

(A) Riccati matrix ODE. `Reason`: Lec. 25 p. 3-4.

(B) Lyapunov matrix ODE.

(C) continuous time algebraic Riccati equation (CARE).

Problem 4

The HJB PDE IVP for continuous time stochastic DP over time horizon $[0, T]$ is

$$\frac{\partial V}{\partial t} + \underset{\boldsymbol{u} \in \mathcal{U}}{\text{minimum}} \left\{ L + \langle \nabla_{\boldsymbol{x}} V, \boldsymbol{f} \rangle + \frac{1}{2} \text{trace}\left(\boldsymbol{\sigma}\boldsymbol{\sigma}^{\top} \text{Hess} V \right) \right\} = 0, \quad V(T, \boldsymbol{x}) = \phi(\boldsymbol{x}).$$

Here ϕ is the

(A) Lagrangian.

(B) Hamiltonian.

(C) terminal cost. `Reason`: Lec. 25 p. 6.

Problem 5

The backward Kolmogorov PDE is a

(A) first order linear PDE.

(B) second order linear PDE. `Reason`: Lec. 25 p. 9.

(C) second order nonlinear PDE.