AerE 5740: Optimal Control: Spring 2026

Iowa State University

Quiz 25: Feynman-Kac Path Integral, Infinite Horizon HJB PDE, Architecture for Partially Observed Optimal Control

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

For any $(t, \boldsymbol{x}) \in [0, T] \times \mathbb{R}^n$, the solution for the backward Kolmogorov PDE IVP

$$\frac{\partial \psi}{\partial t} - q(t, \boldsymbol{x})\psi + \langle \nabla_{\boldsymbol{x}} \psi, \boldsymbol{f}(t, \boldsymbol{x})\rangle + \frac{1}{2}\text{trace}\left(\boldsymbol{\sigma}(t, \boldsymbol{x})\left(\boldsymbol{\sigma}(t, \boldsymbol{x})\right)^{\top} \text{Hess}\, \psi\right) = 0, \quad \psi(t = T, \boldsymbol{x}) = g(\boldsymbol{x}),$$

can be represented as the Feynman-Kac path integral (in the almost sure sense):

$$\psi(t, \boldsymbol{x}) = \mathbb{E}\left[\left(\exp - \int_t^T q\left(s, \boldsymbol{X}_s\right) \mathrm{d}s\right) g\left(\boldsymbol{X}_T\right) \mid \boldsymbol{X}_t = \boldsymbol{x}\right],$$

where $\mathrm{d}\boldsymbol{X}_s = \boldsymbol{f}(s, \boldsymbol{X}_s)\mathrm{d}s + \boldsymbol{\sigma}(s, \boldsymbol{X}_s)\mathrm{d}\boldsymbol{w}_s \; \forall s \in [t, T]$. If g is everywhere ≥ 0, then

(A) the solution $\psi \geq 0 \quad \forall (t, \boldsymbol{x}) \in [0, T] \times \mathbb{R}^n$. `Reason`: product of two nonnegative quantities is nonegative, average of nonnegative is also nonnegative.

(B) the solution $\psi \leq 0 \quad \forall (t, \boldsymbol{x}) \in [0, T] \times \mathbb{R}^n$.

(C) nothing can be guaranteed about the sign of $\psi \quad \forall (t, \boldsymbol{x}) \in [0, T] \times \mathbb{R}^n$.

Problem 2

Give constant $\beta > 0$, the infinite horizon (1 state 1 input) discounted stochastic OCP

$$\min_{\gamma \in \Gamma} \quad \mathbb{E}\int_0^{\infty} e^{-\beta t}\left(\frac{1}{2}x^2 + \frac{1}{2}u^2\right)\mathrm{d}t$$
$$\mathrm{d}x = u\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}w,$$

has DP equation: $\beta V_{\infty} = \underset{u \in \mathbb{R}}{\text{minimum}}\left\{\left(\frac{1}{2}x^2 + \frac{1}{2}u^2\right) + V_{\infty}' u + V_{\infty}''\right\}$, where $'$ denotes derivative with respect to x. The optimal control u_{opt} equals

(A) V_{∞}'.

(B) $-V_{\infty}'$. `Reason`: Take derivative of the expression in curly braces with respect to u and set it equal to zero, to find optimal control.

(C) $-V_{\infty}''$.

Problem 3

With the optimal control from Problem 2, the stationary HJB PDE for that DP equation becomes

$$\beta V_\infty = \frac{1}{2}x^2 - \frac{1}{2}\left(V_\infty'\right)^2 + V_\infty''.$$

In above, substituting a guess $V_\infty(x) = \frac{1}{2}px^2 + c$ for some constants p, c, we find $c = \dfrac{p}{\beta}$ and

(A) $p = \dfrac{-\beta \pm \sqrt{\beta^2+4}}{2}$. `Reason`: Substituting the guess in the DP equation and equating like powers of x gives a quadratic equation in p as $p^2 + \beta p - 1 = 0$, which has this solution.

(B) $p = -\dfrac{\beta}{2}$.

(C) $p = \beta$.

Problem 4

A partially observed LQG control system enjoys

(A) BOTH separation principle and certainty equivalence principle. `Reason`: Lec. 26 p. 14.

(B) ONLY separation principle.

(C) ONLY certainty equivalence principle.

Problem 5

The optimal controller for a partially observed linear Gaussian control system takes the conditional mean of the state as input. This is true for

(A) BOTH quadratic and non-quadratic costs. `Reason`: Lec. 26 p. 13.

(B) ONLY quadratic cost.

(C) ONLY non-quadratic cost.