AerE 5740: Optimal Control: Spring 2025

Iowa State University

Quiz 3: CoV problem formulation

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

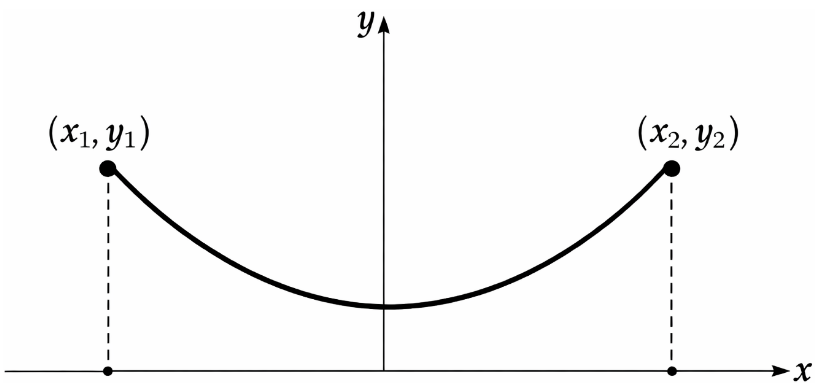

Figure 1: A chain or cable of fixed length ℓ with fixed ends at (x_1, y_1) and (x_2, y_2).

Consider a hanging chain or cable of given length $\ell > 0$ whose ends have given co-ordinates (x_1, y_1) and (x_2, y_2). See Fig. 1. We want to determine the "natural" shape $y = u(x)$ that minimizes the gravitational potential energy $\int (\mathrm{d}m) g y$ where the constant $g > 0$ is the acceleration due to gravity, the differential mass $\mathrm{d}m = \rho \mathrm{d}s$, the constant material density $\rho > 0$, and the differential length $\mathrm{d}s = \sqrt{1 + (u')^2}\,\mathrm{d}x$.

All 5 questions below are for the corresponding CoV formulation

$$\min_{u \in \mathcal{F}(\mathbb{R}) \subseteq \mathcal{C}^1(\mathbb{R})} I(u) = \int_{x_1}^{x_2} L(x, u, u')\,\mathrm{d}x.$$

Problem 1

In this problem, the Lagrangian L equals

(A) $\rho g u$.

(B) $\rho g \sqrt{1 + (u')^2}$.

(C) $\rho g u\,\sqrt{1 + (u')^2}$. `Reason:` substitute $y = u$ and $\mathrm{d}s = \sqrt{1 + (u')^2}\,\mathrm{d}x$ in $\int \rho g y \mathrm{d}s$.

Problem 2

In this problem, the Lagrangian L

(A) has explicit dependence on x.

(B) has NO explicit dependence on x. `Reason:` follows from previous answer.

(C) may have explicit dependence on x depending on problem data $\ell > 0$.

Problem 3

Which of the following describes $\mathcal{C}^1(\mathbb{R})$?

(A) The set of all $u(x)$ continuous in $x \in \mathbb{R}$.

(B) The set of all $u(x)$ differentiable in $x \in \mathbb{R}$.

(C) The set of all $u(x)$ differentiable in $x \in \mathbb{R}$, and $u'(x)$ continuous in $x \in \mathbb{R}$. `Reason:` we explained in class that differentiability is not enough.

Problem 4

In this problem, the constraint set $\mathcal{F}(\mathbb{R})$ equals

(A) $\{u \in \mathcal{C}^1(\mathbb{R}) \mid u(x_1) = y_1, u(x_2) = y_2\}$.

(B) $\{u \in \mathcal{C}^1(\mathbb{R}) \mid u(x_1) = y_1, u(x_2) = y_2, \int_{x_1}^{x_2} \sqrt{1 + (u')^2}\,\mathrm{d}x = \ell\}$. `Reason:` we are given both endpoint and integral constraints.

(C) $\{u \in \mathcal{C}^1(\mathbb{R}) \mid \int_{x_1}^{x_2} \sqrt{1 + (u')^2}\,\mathrm{d}x = \ell\}$.

Problem 5

For this CoV problem to be well-defined, the given data must satisfy

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \leq \ell.$$

The data also need to satisfy

(A) $x_1 \neq x_2$. `Reason:` otherwise the CoV objective is zero and there is nothing to minimize. You can also argue from geometry of the problem.

(B) $y_1 \neq y_2$.

(C) $y_1 = y_2$.