AerE 5740: Optimal Control: Spring 2025

Iowa State University

Quiz 7: EL equation to derive equation-of-motion, introduction to OCP

Name:____

For each of the following multiple choice problems, ONLY ONE option is correct. Clearly mark the correct option. Calculators or other electronic devices are NOT allowed.

Problem 1

Let $c, m > 0$ be constants. In co-ordinate $\boldsymbol{x} \in \mathbb{R}^n$, consider the following Lagrangian with explicit time-dependence:

$$L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) = e^{ct/m} \left(\frac{1}{2} m \langle \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}} \rangle - V(\boldsymbol{x}) \right),$$

where $e^{(\cdot)}$ denotes the exponential function. The corresponding Euler-Lagrange equation $\dfrac{\partial L}{\partial \boldsymbol{x}} = \dfrac{\mathrm{d}}{\mathrm{d}t} \left(\dfrac{\partial L}{\partial \dot{\boldsymbol{x}}} \right)$ gives the equation-of-motion

(A) $m\ddot{\boldsymbol{x}} + c\dot{\boldsymbol{x}} + \dfrac{\partial V}{\partial \boldsymbol{x}} = \boldsymbol{0}$. `Reason:` substitute given L in Euler-Lagrange equation.

(B) $m\ddot{\boldsymbol{x}} + c\dot{\boldsymbol{x}} = \boldsymbol{0}$.

(C) $m\ddot{\boldsymbol{x}} + \dfrac{\partial V}{\partial \boldsymbol{x}} = \boldsymbol{0}$.

Problem 2

For Lagrangian L in Problem 1, the corresponding Hamiltonian $H = \left\langle \dot{\boldsymbol{x}}, \dfrac{\partial L}{\partial \dot{\boldsymbol{x}}} \right\rangle - L$, equals

(A) $\dfrac{1}{2} m \langle \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}} \rangle + V(\boldsymbol{x})$.

(B) $e^{ct/m}$.

(C) $e^{ct/m} \left(\dfrac{1}{2} m \langle \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}} \rangle + V(\boldsymbol{x}) \right)$. `Reason:` substitute given L in given expression for H.

Problem 3

For a continuous-time OCP over time interval $[0, T]$, the terminal cost ϕ

(A) depends only on final time T and final state $\boldsymbol{x}(T)$. `Reason:` Lec. 8, p. 11, 13.

(B) depends only on initial state $\boldsymbol{x}(0)$.

(C) depends only on terminal constraint.

Problem 4

For a continuous-time OCP, the integrand of the cost-to-go is called

(A) the Lagrangian. `Reason:` Lec. 8, p. 11.

(B) the Hamiltonian.

(C) the controlled dynamics.

Problem 5

The controlled dynamics $\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$, $\boldsymbol{x} \in \mathbb{R}^{10}$, $\boldsymbol{u} \in \mathbb{R}^2$, is

(A) 2×1 vector first order ODE.

(B) 10×1 vector first order ODE. `Reason:` Left-hand-side has size 10×1 because $\boldsymbol{x} \in \mathbb{R}^{10}$.

(C) 12×1 vector first order ODE.